The spectrum of the universe of codes

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Abstract

We investigate the spectrum of elementary particles and fields arising from the superposition of string configurations weighted according to their entropy in the string phase space. We find that this superposition describes a universe with a physical content phenomenologically compatible with the experimental observations and measurements. Masses and couplings are determined as functions of the age of the universe, with no room for freely-adjustable parameters. They depend on time, with a scaling allowing this scenario to pass the tests provided by cosmology and the constraints imposed by the physics of the primordial universe.

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Contents

1 Introduction

In Ref. [1] we have discussed a theoretical scenario in which the universe is given by the superposition of all possible configurations which describe the assignment of a certain amount of energy along a vector space, of any possible dimension. The time ordering of the history of the universe is given by the inclusion of the sets containing all the configurations at a certain total energy. All the information about the universe at total energy $\mathcal{E}$ is encoded in a "partition function" that can be expressed as:

$$\mathcal{Z}_{\mathcal{E}} \;=\; \sum_{\psi(E\leq\mathcal{E})} \mathrm{e}^{S(\psi)}\,, \tag{1.1}$$

where $\psi(E)$ indicates a configuration (i.e. a distribution of E energy units along space), and $S(\psi)$ is the entropy, i.e. the logarithm of the volume of occupation of the configuration ψ in the phase space of all the possible configurations. The dominant configuration of the universe is the one of highest entropy. The contribution of all the neglected configurations falls under the error accounted for by the Heisenberg Uncertainty; indeed, 1.1 implies a quantum scenario which also embeds special and general relativity, and therefore quantum gravity. In Ref. [2] we identified in String Theory the theory which realizes a representation of this scenario, i.e. the evolution of the superposition of geometries, in terms of propagating fields. We discussed how free fields and particles can be investigated in a limit in which gravity is decoupled and these degrees of freedom appear to propagate in a flat space. This corresponds to a logarithmic representation of the theory, i.e. on the tangent space, where groups and their multiplicative properties are approximated by algebras and sums instead of products. The analogous of 1.1 on the continuum is:

$$\mathcal{Z}_{V} \;=\; \int_{V} \mathcal{D}\psi\, \mathrm{e}^{S(\psi)}\,, \tag{1.2}$$

where now ψ indicates a string configuration, and $S(\psi)$ its volume in the phase space of all string configurations at finite target-space volume V, measured in the duality-invariant Einstein's frame.

Object of this paper is the investigation of the properties of elementary particles and fields. According to 1.2, in first order they are obtained by looking at the most entropic string configurations. In our theoretical framework string configurations are in themselves not backgrounds but full configurations of the universe, which already contain in the shape of their geometry the field and particle perturbing the ground geometry. It is only when they are perturbatively "flattened" that the string configurations appear as "vacua", background geometries on which field and particle excitations leave, as in the ordinary approach to string theory. Investigating the configurations of highest entropy in the limit of flat geometry tells us about the spectrum of the theory, i.e. which particles and fields we must consider as the elementary excitations which propagate in the physical universe. Since the fundamental coupling of the theory is the gravitational coupling, by definition set to one being the unit of scale (the Planck scale unit), the full theory is basically strongly coupled, and the investigation of its properties can only be performed through comparison of a full bunch of string dual constructions. Moreover, since the universe is the result of a superposition of configurations, the properties of the spectrum of elementary particles too are not associated to one single configuration, but result from the superposition of configurations. There is no unique string vacuum which can be singled out as "the" right configuration, containing all the physics we want. In particular, there is no single string configuration in which one can observe the differentiation in the mass spectrum of elementary particles. The difference in the masses of particles belonging to different families originates from the fact that families have a different weight in the phase space of string configurations, i.e. they do not appear on the same number of configurations; the hierarchy of masses reflects the hierarchy of their occurrence. Similarly it goes for any other symmetry: the fact of being the universe a superposition of configurations with different degree of symmetry eventually produces a universe in which any symmetry is broken. The lower is the difference in the weights of configurations with different symmetry, the softer results to be the breaking of the symmetry.

As to be espected from the discussion in Ref. [2] the sum 1.2 turns out to imply the three-dimensionality of space: in the configurations of highest entropy only a four-dimensional subspace is allowed to expand, and indeed, owing to the presence in the spectrum of massless fields, it expands; all the remaining coordinates are twisted. As expected, supersymmetry is broken at the Planck scale. The spectrum of the elementary excitations corresponds to the degrees of freedom of all the known elementary particles, and their interactions. Despite the lack of low-energy supersymmetry, the cosmological constant is correctly predicted without fine tuning because the string vacuum energy expectation value, in our case of order one [1], does not correspond to an energy density, but to a quantity that, in order to be transformed into a density, must be rescaled by a Jacobian accounting for the coordinate transformation from the string to the Einstein's frame; this introduces a suppression corresponding to a two-volume, the square of the radius of space-time. The so produced true density is therefore $\Lambda \sim 1/R^2 \sim H^2$, where H is the Hubble constant [2]. It turns out to be not at all a constant, evolving with the inverse square of the age of the universe. This reproduces what derived in the original formulation of the scenario, Ref. [1].

The mass of a particle arises as its ground momentum in a compact space. The size of the mass depends on the weight of the configuration containing this particle in the phase space of all the configurations (i.e. it depends on the "multiplicity" of the momentum). This means that it will be some fractional power of the fundamental momentum, and therefore of the order of some root of the inverse of the age of the universe. The result is that all masses depend on time. At present, their values can be seen to agree with the experimental ones. The effect of the time-variation of masses can be observed in the time-variation of the cosmic emission spectra, causing the apparent acceleration of the expansion of the universe, and in other deviations of astrophysical observations from what expected on the basis of present-day parameters. As according to Ref. [2] also couplings are related to volumes in the phase space, and here are derived by comparing the ratios of volumes of the symmetry groups of the most entropic string configurations. Since the relative weights of the configurations depend on the size of the whole phase space at a certain age of the universe, like masses also couplings turn out to depend on the age of the universe. Despite the absence of supersymmetry, they naturally unify at the Planck scale: their running on the large scale is in fact here not related to the logarithmic running we use in an effective action of the elementary particles.

Although almost any physically observable quantity receives a different explanation than

[1] Indeed it is precisely 1 by choice of normalization of the mean values.

[2] The basic absence of invariance under space-time translations implies, by construction, a different normalization of string amplitudes, and therefore a different interpretation of the computed mean values: owing to the absence of a normalization factor $1/\mathcal{V}$, where $\mathcal{V}$, the four-volume of space, corresponds to the volume of the group of translations, densities are now lifted to global quantities.

in traditional field theory or cosmology approaches, it is nevertheless consistent with what is experimentally measured. Indeed, precisely the high predictive power of this theoretical scenario, due to the fact that there are no free parameters that can be adjusted in order to fit data, enhances the strength of any matching with experimental results: any discrepancy could in fact rule out the entire construction. Because of this, a large part of the investigation is devoted to re-analyzing the most important data and constraints, coming not only from elementary particles physics but also from astrophysics and cosmology. Also here, within the degree of approximation introduced in the computation predictions and results are compatible with the experimental data.

Among the *highlights* of the predictions, for the elementary particle physics we mention here:

- - correct prediction of the present-time value of masses and couplings;
- - no low-energy supersymmetry (till the Planck scale);
- - no new *elementary* particles;
- - no Higgs mechanism to give rise to masses;
- - new resonances and possibly non-local correlation effects in LHC physics.

Indeed, the quantum mechanics arising from 1.1 implies departures from the quantum behavior expected in the traditional models of electro-dynamics and weak interactions, especially in relativistic systems characterized by a high geometric complexity. These effects can be viewed as due to the quantization of space, as implied in a quantum gravity scenario. When one thinks at the contribution of gravity, one usually has in mind the contribution of *classical* gravity, and thinks at quantization as something that involves only the description of the graviton as a quantum field. Indeed, quantum gravity implies much more: it concerns the quantization of the geometry, and its effects show up in any complex system, even in conditions in which classical gravity can be neglected. For instance, as discussed in Ref. [3], it allows to explain the correlation between complexity of the lattice structure and critical temperature in high-temperature superconductors. Similarly, it predicts a higher degree of non-locality of wave-functions in quantum systems at high energy. For instance, this should show out in a higher degree of correlation among products of a high energy collision [3].

For what matters the large time-scale and cosmology we list then the following highlights of this scenario:

- - time-dependence of masses and couplings;
- - correct prediction of the present-time value of the cosmological constant;
- - explanation of cosmological observations without dark matter;
- - compatibility with cosmological constraints like nucleosynthesis and Oklo bound.

[3] It indeed seems that effects of this kind are being detected at LHC.

For each of these phenomena the explanation relies in the particular evolution of mass scales and couplings, as functions of the age of the universe. As discussed in Ref. [4], this scenario seems to give also a correct prediction of the magnitude of CP violation not only in the case of K-meson system, but also for the much more problematic D-meson system.

1.1 Outline of the work

The paper, in large part an update of [5], is organized as follows. We start in section 2 by investigating the string phase space through orbifold constructions. The highest entropy in the string phase space is attained with the highest amount of twists and shifts. The maximum is not sharply picked around one single construction, but is spread around a small bunch of configurations which differ by the action of some shifts. Their stapling gives rise to the mass differentiation of the spectrum. We re-derive the result of [1] about the number of dimensions of space-time, here identified with the number of coordinates which remain untwisted, and therefore free to expand. We review also the discussion of [1] about the number of dimensions in which the non-perturbative string theory lives, in the light of non-perturbative string-string dualities between orbifold constructions (section 2.1). Knowing the whole number of dimensions will turn out useful in order to compute the neutron mass in section 4.3.6. We discuss then the origin of masses, the spectrum of the theory, and the breaking of the symmetry within and between families of particles. Finally we comment the issues related to the magnetic monopoles, in particular the topological ones, which in this scenario are expected to not exist.

In section 3 we consider the geometry of the universe and the type of expansion it undergoes. We pass then in section 4 to the detailed derivation of the couplings, the masses of the elementary particles and of the massive bosons, as functions of the age of the universe. We derive then the scaling of the mean mass scale of the universe, a quantity that can be non-perturbatively computed in an exact way: it corresponds in fact to the only eigenvalue of the Hamiltonian at any finite space-time volume. This scale can be seen to basically correspond to the mass of stable matter: if the matter present in the universe were constituted by particles all of the same kind, these would have a mass precisely corresponding to this scale. This scale can be shown to roughly correspond to the neutron mass. With the scaling of the average mass of the universe at hand we can discuss the issue of the apparent acceleration of the universe, and show that in this scenario of non-accelerated expansion the observed acceleration of red-shifts can be explained as due to the variation with time of the atomic energy levels, and consequently of emitted light. In particular, we comment this point of view in comparison to the usual approach to the acceleration of the expansion of the universe.

In section 5 we come to the explicit evaluation of masses and couplings at present time. In particular, in section 5.4 we compute the fine structure constant (indeed its present-day, value because in our case it is not a constant), obtaining a value which falls within an error of $\sim 5 \times 10^{-6}$ away from its most updated experimental value. In sections 5.5–5.7 we briefly discuss also baryon and meson masses. Their values too agree, within the approximations introduced in the computing procedure, with what experimentally observed. In section 5.8 we discuss the mass of the gauge bosons of the weak interactions, and how the terms of this sector of the Standard Model are effectively reproduced.

The investigation of the mass sector of the theory is completed in section 6, where we consider the mixing angles of weak decays (the Cabibbo-Kobayashi-Maskawa matrix). Since in our scenario neutrinos are massive, mixing of generations and off-diagonal decays are expected to occur also among leptons. Differently from the previous versions of this work, CP violation is now discussed in a separated paper entirely devoted to this phenomenon, Ref. [4].

In section 7 we go a step beyond the spectrum of free elementary particles, to investigate the interacting theory; in particular, we discuss how in the full, non-perturbative theory there are interactions and resonances that can not be predicted in the ordinary perturbative approach. According to our point of view, also the resonace around 125 GeV recently detected at LHC, commoly interpreted as a Higgs boson signal, is of this kind.

We consider then the "Cosmic Microwave Background" radiation, and discuss how in this framework the existence of a $\sim 2,8\,^0$ Kelvin radiation comes out as a prediction. We also discuss, in subsection 7.3, the case of dark matter. In our scenario, this is expected to not exist. We comment several cases which are usually considered to provide evidence for its existence, and propose how, within our framework, in each of them the effects attributed to dark matter receive an alternative explanation. In section 7.4 we discuss then the constraints on the evolution of masses and couplings coming from the observation of ancient regions of the universe, or, as is the case of the Oklo bound, from the history of our planet. We find out that the predicted behavior is compatible with all the constraints. Not only, but in the case of the so-called "time dependence of α", it turns out to correctly predict the magnitude of the observed effect (section 7.4.1).

2 The non-perturbative solution

The integral 1.2 contains in principle all the information about our universe. As discussed in [2], the main contribution to the appearance of the universe is given by the configurations of minimal symmetry, because they have at any time the highest entropy in the phase space at fixed volume. In order to investigate the physical content of the theory we will use a "perturbative" approach. In ordinary quantum field theory one separates the time evolution into a free propagation and an interaction part. The physical configurations are inspected via the conceptual separation of a base of free states, eigenstates of the free Hamiltonian, which are exact solutions of the free theory. As long as the coupling of the interaction is small, the full solution can be considered a small perturbation of the free propagation, and the perturbative approach makes sense. In our case, we have a truly non-perturbative string system, in which even the space-time is mixed up, and in general will not be factorisable into an extended one, "the" space-time as we experience it, and an internal space. Moreover, we can access the whole theory only through "slices", the perturbative (string) constructions, to be treated as the patches, the "projections", which allow to shed light into the "patchwork", the whole theory. We will get information about the true vacuum through heavy use of string-string duality, and, consistently with the fact that we are investigating a flat limit of the geometry, we will follow the process of symmetry reduction through the spectrum of possible string constructions in the class of orbifolds. Orbifolds are particular string constructions in which the target space is flat everywhere except from some special points, at which the curvature is concentrated. Having full knowledge of the spectrum of the perturbative states at any energy level, we are able to write the partition function, the "one loop partition function", which in principle encodes all the information about the construction; with this it is possible to explicitly perform one-loop computations of scattering amplitudes and threshold corrections, and therefore compare string duals through pure string computations. Z_2 orbifolds are the best suited for our investigation, because they preserve the basic structure of the target space as a product of circles (it becomes a generic product of circles and orbifolded circles, S^1/Z_2) and mod-out the space by the group with the smallest volume among all the orbifold operations. A product of Z_2 twist/shifts allows therefore to achieve a configuration whith a smaller surviving symmetry group than those obtained through any other product of orbifold operations. Entropy will therefore be the maximal we can obtain with orbifold operations. The most entropic orbifold vacuum will be the one with the highest amount of freely and non-freely acting Z_2 shifts and twists. Unfortunately, this configuration can be constructed explicitly only in a perturbative regime. This corresponds to the decompactification of some coordinate which serves as coupling of the theory. As a consequence, we will never see explicitly all the properties of the whole theory: these can only be indirectly inferred through the comparison of dual constructions. In particular, the amount of supersymmetry will appear in a different way, depending on whether the decompactified coordinate does also tune the supersymmetry breaking, or not.

2.1 *Investigating orbifolds through string-string duality*

Investigating the non-perturbative properties of a string vacuum by comparing dual constructions is neither an easy task, nor a straightforward one. In general, at a generic point in the moduli space the full set of dual constructions, enabling to "cover" the full content, is not known. Some progress on its knowledge has been done in the case of supersymmetric vacua with extended supersymmetry, where it is in general possible to identify a subset of the spectrum made "stable" by the properties of supersymmetry. The case of orbifolds turns out to be particularly suited for the investigation of non-perturbative string-string dualities. In this case it is possible to make a non-trivial comparison of the renormalization of terms that receive contributions only from the so called BPS states, and this not just on the ground of the properties of supersymmetry, but through the computation of true string contributions. Fortunately, Z_2 orbifolds, the case of our interest, are the easiest and therefore more investigated constructions [4]. Indeed, through the analysis of these constructions, it is possible to get an insight into the properties which are typical of string theory in itself: most of the investigations performed at other points in the moduli space must in fact rely on geometrical properties of smooth surfaces, and their singularities. Although for some respects rather powerful, these techniques don't allow to capture the presence of states related to non-geometrical singularities, or even fail in general for the simple reason that, owing to T-duality, the full string space simply cannot be reduced to a geometrical one[5].

Our starting point is a maximally supersymmetric string vacuum with flat background given by a product of circles. The constraints of two-dimensional conformal field theory impose that Z_2 orbifold twists must act on groups of four coordinates at once. In any string construction, there is room for a maximum of 3 such operations, one of which is however redundant, in that it leads, once combined with the other ones, to the re-introduction in the twisted sectors of the states projected out. Therefore, we can say that only a maximum of two independent Z_2 twists act effectively. However, the amount of supersymmetry surviving to these projections, as well as the amount of initial supersymmetry, is different, depending on whether we start with heterotic, type I, or type II strings. This means that in any construction not all the projections acting on the theory are visible. Indeed, one of them is always non-perturbative. The reason is that, by definition, a perturbative construction is an expansion around the zero value of a parameter, the coupling of the theory, which is itself a coordinate in the whole theory. An orbifold operation acting on this coordinate is forcedly non-perturbative [6]. In the following we will often make use of the language of string compactifications to four dimensions, especially for what matters our reference to the moduli of the string orbifolds. This will turn out to be justified "a posteriori": we will see that indeed the final configuration is the one of a string space with all but four coordinates twisted and therefore "frozen". Only four coordinates remain un-twisted and free to expand, while all the others remain stuck at the string/Planck scale. Massless degrees of freedom

[4] See for instance Refs. [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35].

[5] For examples, see for instance Ref. [17].

[6] A first investigation of a non-perturbative orbifold, which produces the heterotic string, has been carried out in [36, 37].

move along these and expand the horizon of space-time at the speed of light. Although not infinitely extended, this "large" space is what in our scenario corresponds to the ordinary space-time. The language of orbifold constructions in four dimensions is therefore just an approximation, that works particularly well at large times. Only at a second stage, we will also discuss how and where this picture must be corrected in order to account also for compactness of the space-time coordinates. Although somehow an abuse of language, this approximation allows us to take and use with little changes many things already available in the literature. In particular, for several preliminary results and a rediscussion of the previous literature, the reader is referred to Ref. [17].

Let's see what are in practice the steps of decreasing symmetry we encounter when approaching the most singular configuration. Although at the end it will be irrelevant the order in which we apply freely and non-freely acting orbifold operations, it is convenient to organize the analysis by considering first non-freely acting operations, i.e. pure twists with orbifold fixed points. Starting from the M-theory configuration with 32 supercharges, we come, through orbifold projections, to 16 supercharges and a gauge group of rank 16. Further orbifolding leads then to 8 supercharges ($\mathcal{N}_4 = 2$) and introduces for the first time non-trivial matter states (hypermultiplets). As we have seen in [17] through an analysis of all the three dual string realizations of this vacuum (type II, type I and heterotic), this orbifold possesses three gauge sectors with maximal gauge group of rank 16 in each. The matter states of interest for us are hypermultiplets in bi-fundamental representations: these are in fact those which at the end will describe leptons and quarks (all the others are eventually projected out). As discussed in [17], in the simplest formulation the theory has 256 such degrees of freedom. The less symmetric configuration is however the one in which, owing to the action of further Z_2 shifts, the rank is reduced to 4 in each of the three sectors. These operations, acting as rank-reducing projections, have been extensively discussed in [38, 13, 14, 17]. The presence of massless matter is in this case still such that the gauge beta functions vanish. In this case, the number of bi-charged matter states is also reduced to $4 \times 4 = 16$. These states are indeed the twisted states associated to the fixed points of the projection that reduces the amount of supersymmetry from 16 to 8 supercharges.

Let's consider the situation as seen from the type II side. We indicate the string coordinates as $\{x_0, \ldots, x_9\}$, and consider $\{x_0, x_9\}$ the two longitudinal degrees freedom of the light-cone gauge. The transverse coordinates are $\{x_1, \ldots, x_8\}$. Here all the projections appear as left-right symmetric. The identification of the degrees of freedom, via string-string duality, on the type I and heterotic side depends much on the role we decide to assign to the coordinates, as we will see in a moment. By convention, we choose the first Z_2 to twist $\{x_5, x_6, x_7, x_8\}$:

$$Z_2^{(1)}: \quad (x_5, x_6, x_7, x_8) \to (-x_5, -x_6, -x_7, -x_8)\,, \tag{2.1}$$

and the second Z_2 to twist $\{x_3, x_4, x_5, x_6\}$:

$$Z_2^{(2)}: \quad (x_3, x_4, x_5, x_6) \to (-x_3, -x_4, -x_5, -x_6)\,. \tag{2.2}$$

These two projections induce a third one: $Z_2^{(1,2)} \equiv Z_2^{(1)} \times Z_2^{(2)}$, that twists $\{x_3, x_4, x_7, x_8\}$:

$$Z_2^{(1,2)}: \quad (x_3, x_4, x_7, x_8) \to (-x_3, -x_4, -x_7, -x_8)\,. \tag{2.3}$$

Altogether, they reduce supersymmetry from $\mathcal{N}_4 = 8$ to $\mathcal{N}_4 = 2$, generating 3 twisted sectors. Depending on whether we consider the type IIA or IIB construction, the twisted sectors give rise either to matter states (hyper-multiplets) or to gauge bosons (vector-multiplets). As we discussed in Ref. [17], a comparison with the heterotic and type I duals shows that the underlying theory must be considered as the union of the two realizations: owing to the lack of a representation of vertex operators at once perturbative for all of them, for technical reasons no one of the constructions is able to explicitly show the full content of this vacuum. The matter (and gauge) content in these sectors is then reduced by six Z_2 shifts acting, two by two, by pairing each of the three twists of above with a shift along one of the two coordinates of the set $\{x_1, \ldots, x_8\}$ which are not twisted. Each shift reduces the number of fixed points of a Z_2 twist by one-half; two shifts reduce therefore the matter states of a twisted sector from 16 to 4. Altogether we have then, besides the $\mathcal{N}_4 = 2$ gravity supermultiplet, three twisted sectors giving rise each one to 4 matter multiplets (and a rank 4 gauge group). On the type I side, these three sectors appear as two perturbative D-brane sectors, D9 and D5, while the third is non-perturbative. On the heterotic side, two sectors are non-perturbative. As it can be seen by investigating duality with the type I and heterotic string, the matter states from the twisted sectors are actually bi-charged (see Refs. [39, 40], and [17]), something that cannot be explicitly observed, the charges being entirely non-perturbative from the type II point of view. The moduli $T^{(1)}$, $T^{(2)}$, $T^{(3)}$ of the type II realization, associated respectively to the volume form of each one of the three tori $\{x_3, x_4\}$, $\{x_5, x_6\}$, $\{x_7, x_8\}$, are indeed "coupling moduli", and correspond to the moduli "S", "T", "U" of the theory. On the heterotic side, S is the field whose imaginary part parametrizes the string coupling: $\mathrm{Im}\, S = \mathrm{e}^{-2\phi}$. It is therefore the coupling of the sector that contains the gravity fields. T and U are perturbative moduli, and correspond to the couplings of the two non-perturbative sectors. On the type I side, on the other hand, two of them are non-perturbative, coupling moduli, respectively of the D9 and D5 branes, while only one of them is a perturbative modulus, corresponding to the coupling of a non-perturbative sector [39, 41, 42, 28]. Owing to the artifacts of the linearization of the string space provided by the orbifold construction, gravity appears to be on a different footing on each of these three dual constructions.

2.1.1 The maximal twist

The configuration just discussed constitutes the last stage of orbifold twists at which we can "easily" follow the pattern of projections on all the three types of string construction. It represents also the maximal degree of Z_2 twisting corresponding to a supersymmetric configuration. As we will see, a further projection necessarily breaks supersymmetry. The vacuum appears supersymmetric only in certain dual phases, such as the perturbative heterotic representation. Non-perturbatively, supersymmetry is on the other hand broken. This means that, when further twisted, the theory is basically no more de-compactifiable: perturbative, i.e. decompactification, phases, represent only approximations in which part of the theory content and properties are lost, or hidden. This is what usually happens when one for instance pushes to infinity the size of a coordinate acted on by a Z_2 twist. The situation is the one of a "non-compact orbifold".

The further Z_2 twist we are going to consider is also the last that can be applied to this vacuum, which in this way attains its maximal degree of Z_2 twisting. This operation, and the configuration it leads to, appears rather differently, depending on the type of string approach. Let's see it first from the heterotic point of view. So far we are at the $\mathcal{N}_4 = 2$ level. The next step appears as a further reduction to four supercharges (corresponding to $\mathcal{N}_4 = 1$ supersymmetry). Of the previous projections, $Z_2^{(1)}$ and $Z_2^{(2)}$, only one was realized explicitly on the heterotic string, as a twist of four coordinates, say $\{x_5, x_6, x_7, x_8\}$. The further projection, $Z_2^{(3)}$, acts on another four coordinates, for instance $\{x_3, x_4, x_7, x_8\}$. In this way we generate a configuration in which the previous situation is replicated three times. When considered alone, the new projection would in fact behave like the previous one, and produce two non-perturbative sectors, with coupling parametrized by the moduli of a two-torus, in this case $\{x_5, x_6\}$: $T^{(5-6)}$, $U^{(5-6)}$. The product $Z_2^{(1)} \times Z_2^{(3)}$ leaves instead untwisted the torus $\{x_7, x_8\}$ and generates two non-perturbative sectors with couplings parametrized by the moduli $T^{(7-8)}$, $U^{(7-8)}$. Altogether, apart from the projection of states implied by the reduction of supersymmetry, the structure of the $\mathcal{N} = 2$ vacuum gets triplicated.

The symmetry of the action of the additional projection with respect to the previous ones suggests that the basic structure of the configuration, namely its repartition into three sectors, S, T, U, is preserved when passing to the less supersymmetric configuration. This phenomenon can be observed in the type II dual, that we discuss in detail in Appendix B. From the heterotic point of view, the states of these sectors come replicated ($\{T\} \to \{T^{(3-4)}, T^{(5-6)}, T^{(7-8)}\}$, $\{U\} \to \{U^{(3-4)}, U^{(5-6)}, U^{(7-8)}\}$). On the type II side we observe a triplication also of the "S" sector. However, as we discussed in Ref. [17], we are faced here to an artifact of the orbifold constructions, that by definition are built over a linearization of the string space into planes separated by the orbifold projections. The matter states are indeed charged under three sectors, S^i, T^j, U^k, but we can at most observe a double charge, as it appears on the type I dual side; from an analysis based on string-string duality, we learn that the states are in fact multi-charged for mutually non-perturbative sectors When one of the S^i, T^j, U^k sectors is at the weak coupling, the other two are at the strong coupling, and it doesn't make sense to ask what is this sort of "splitting" of the non-perturbative charge of the states: we simply observe that they have a perturbative index and one running on a strongly coupled part of the theory. On the type I dual realization of this vacuum, besides a D9 branes sector we have now three D5 branes sectors and a replication of the non-perturbative sector into three sectors, whose couplings are parametrized by $U^{(3-4)}$, $U^{(5-6)}$, $U^{(7-8)}$.

A result of the combined action of these projections is that all the fields S^i, T^j and U^k are now twisted. This means that their vacuum expectation value is not anymore running, but fixed at a scale to be identified with the string-string duality-invariant Planck scale. Nevertheless, for convenience here we continue with the generic notation S, T, U used so far, because it allows to better follow the functional structure of the configuration we are investigating. Twisting of the "coupling" moduli indeed suggests the non-decompactifiability of this vacuum. This, as discussed, would imply the breaking of supersymmetry. However, this property is not so directly evident: each dual construction is in fact by definition perturbatively constructed around a decompactification limit. The point is to see, with the help of string-string duality, whether this is a real decompactification, or just a singular, non-compact orbifold limit. An important argument in favour of this second situation is that, after the $Z_2^{(3)}$ projection is applied, the so-called "$\mathcal{N} = 2$ gauge beta-functions" are unavoidably non-vanishing. According to the analysis of Ref. [17], this means that there are hidden sectors at the strong coupling [7]. As a consequence, supersymmetry is actually non-perturbatively broken by gaugino condensation. Inspection of the type II string dual shows explicitly the instability of the $\mathcal{N}_4 = 1$ supersymmetric vacuum.

In order to construct the type II dual, it is not possible to proceed as with the heterotic and type I string, namely by keeping un-twisted some coordinates. On the type II side the "$\mathcal{N}_4 = 1$" vacuum looks rather differently: the new projection twists all the transverse coordinates, leaving no room for a "space-time". This however does not mean that a space-time does not exist: all non-twisted coordinates, therefore the space-time indices, are non-perturbative. Their volume is precisely related to the size of the coupling around which the perturbative vacuum is expanded. After $Z_2^{(1)}$ and $Z_2^{(2)}$, the only possibility for applying a perturbative Z_2 twist is in fact to act on $\{x_1, x_2, \}$ and on two of the $\{x_3, \ldots, x_8\}$ coordinates, already considered by the previous twists. These can be either the pair $\{x_3, x_4\}$ or $\{x_5, x_6\}$, or $\{x_7, x_8\}$. Which pair, is absolutely equivalent. We can chose $Z_2^{(3)}$ such that:

$$Z_2^{(3)}: \quad (x_1, x_2, x_3, x_4) \ \to \ (-x_1, -x_2, -x_3, -x_4)\,. \tag{2.4}$$

The other choices are anyway generated as $Z_2^{(3)} \times Z_2^{(2)}$ and $Z_2^{(3)} \times Z_2^{(2)} \times Z_2^{(1)}$. Assigning a twist to some coordinates is not enough in order to define an orbifold operation: the specification must be completed by an appropriate choice of "torsion coefficients". The analysis of this orbifold turns out to be easier at the fermionic point, where the world-sheet bosons of the conformal theory are realized through pairs of free fermions [43]. We leave to the appendix B a detailed discussion of the construction of this vacuum. There we see how the duality map with $\mathcal{N} = 2 \to \mathcal{N} \to 1$ heterotic theory imposes a choice of "GSO coefficients" that leads to the complete breaking of supersymmetry. Since the breaking is tuned by moduli which at the highest level of symmetry breaking (i.e. at the highest entropy configurations) are twisted, supersymmetry is broken at a scale which is eventually identified with the Planck scale [8].

The reason why the breaking of space-time supersymmetry can be observed in a dual in which space-time is entirely non-perturbative relies on the unambiguous identification of the supersymmetry generators. More precisely, what on the type II side it is possible to see is the projection of the supersymmetry currents on the type II perturbative space. Target space supersymmetry is in fact realized in string theory through a set of currents whose representation is built out of the world-sheet degrees of freedom. For instance, in the case

[7] We refer the reader to the cited work for a detailed discussion of this issue.

[8] Among the historical reasons for the search of low-energy supersymmetry are the related smallness of the cosmological constant and the stabilization in the renormalization of mass scales produced by supersymmetry. In our framework, the value of the cosmological constant will be justified in a completely different way (section 3). Also the issue of stabilization of scales in this framework must be considered in a different way: masses are no more produced by a field-theory mechanism, and field theory is not the environment in which to investigate their running.

of free fermions in four dimensions, we have:

$$G(z) \;=\; \partial_z X^\mu \psi_\mu \,+\, \sum_i x^i y^i z^i \,, \tag{2.5}$$

and

$$G(\bar z) \;=\; \partial_{\bar z} X^\mu \bar\psi_\mu \,+\, \sum_i \bar x^i \bar y^i \bar z^i \,, \tag{2.6}$$

where the index i runs over the internal dimensions. At the $\mathcal{N}_4 = 2$ level it is possible to construct both the representations of the type II dual, namely the one in which space-time is perturbative, and the one in which space-time is non-perturbative., Tracing the representation of the supersymmetry currents in both these pictures allows us to identify them also when the $Z_2^{(3)}$ twist is applied. Although, strictly speaking, there is no simple one-to-one linear mapping between coordinates of dual constructions, the fact that the dual representations of the currents share a projection onto a subset of coordinates common to both, enables us to follow the fate of space-time supersymmetry anyway.

The analysis of the type II dual confirms that the matter states of this vacuum are indeed three replicas of the chiral fermions of the theory before the supersymmetry-breaking, $Z_2^{(3)}$ projection. In the type II construction their space-time spinor index runs non-perturbatively; they appear therefore as scalars. In total, we have three sets of bi-charged states in a $\mathbf{16} \times \mathbf{16}$. In the minimal, semi-freely acting configuration, they get reduced to three sets of $\mathbf{4} \times \mathbf{4}$ by the further Z_2 shifts, acting on the twisted planes. As it was the case of the $\mathcal{N}_4 = 2$ theory, on the type II side their charges are non-perturbative, and they misleadingly appear as $(\mathbf{16}, \mathbf{16}, \mathbf{16})$, reduced to $(\mathbf{4}, \mathbf{4}, \mathbf{4})$. The impression is that we have three families of three-charged states. However, this is only an artifact of the orbifold construction. From the heterotic point of view, namely, the vacuum in which gauge charges are visible, two sectors of each family are non-perturbative and, as previously mentioned, the structure of their contribution to threshold corrections is an indirect signal that they are at the strong coupling (see Ref. [17]). The situation is the following: either 1) we explicitly see all the gauge sectors, on the type II side, but we don't see the gauge charges, or 2), in the constructions in which we can explicitly construct currents and see gauge charges (the heterotic realization), we see the gauge sector, and the currents, corresponding to just one index born by the matter states, whereas the other ones are non-perturbative and strongly coupled.

The type II realization appears to be a different "linearization", or linear representation, of the string space, in which the non-perturbative curvature has been "flattened" through an embedding in a higher number of (flat) coordinates, which goes together with a redundancy of states due to an artificial replication of some degrees of freedom. On the type II string, twisted states can only be represented as uncharged, free states. Their charges are in any case non-perturbative, and we cannot observe a "non-abelian gauge confinement". These gauge sectors appear as partially perturbative on the type I side. However, the type I vacuum, like the heterotic one, corresponds to an unstable phase of the theory: it appears as supersymmetric although it is not. Moreover, inspection of the gauge beta-functions reveals that they are positive. Therefore, although appearing as free states, the states on the D-branes run to the strong coupling and the apparent gauge symmetries are broken by confinement.

Let's **summarize** the situation. The initial theory underwent three twists and now is essentially the following orbifold:

$$Z_2^{(1)} \times Z_2^{(2)} \times Z_2^{(3)} \,. \tag{2.7}$$

In terms of supercharges, the supersymmetry breaking pattern is:

$$32 \xrightarrow{Z_2^{(1)}} 16 \xrightarrow{Z_2^{(2)}} 8 \xrightarrow{Z_2^{(3)}} 0 \ \ (4 \text{ only perturbatively}) \,. \tag{2.8}$$

The "twisted sector" of the first projection gives rise to a non-trivial, rank 16 gauge group; the twisted sector of the second leads to the "creation" of one matter family, while after the third projection we have a replication by 3 of this family. The rank of each sector is then reduced by Z_2 shifts of the type discussed in Ref. [13, 10, 14], two per each complex plane. As a result, each **16** is reduced to **4**. On the type II side one can explicitly see, besides the shifts, both the total breaking of supersymmetry and the doubling of sectors under which the matter states are charged. The product of these operations leads precisely to the spreading into sectors that at the end of the day separate into weakly and strongly coupled, allowing us to interpret the matter states as quarks [9]. On the type I side, the states appear in an unstable phase, as free supersymmetric states of a confining gauge theory, while on the heterotic side they appear on the twisted sectors, and their gauge charges are partly non-perturbative, partly perturbative. The perturbative part is realized on the currents. Like the type I realization, also the heterotic vacuum appears to be an unstable phase, before flowing to confinement; both are indeed non-perturbatively singular, non-compact orbifolds. This reflects on the fact that, as also discussed in Ref. [17], both on the heterotic and type I side, perturbative and non-perturbative gauge sectors have opposite sign of the beta-function. This signals that, as the visible phase is confining, the hidden one is non-confining. The matter states of the theory consist therefore of a replica into three families of a bi-charged complex state transforming as $\mathbf{4^w} \times \mathbf{4^s}$, where the $\mathbf{4^w}$ belongs to a weakly coupled sector, while the $\mathbf{4^s}$ to a strongly coupled sector of the theory. Indeed, the fact that 1) with the last twist supersymmetry is broken, 2) the internal string space is curved, and 3) the coupling does not correspond anymore to a modulus but is twisted, frozen at a value of order one in (duality-invariant) Planck units, means that the theory in itself is at the strong coupling, and that a perturbative realization is only possible as a projection onto some subsectors. After further symmetry breaking the $\mathbf{4^w}$ will give rise to the weak interactions, while the $\mathbf{4^s}$ to the strong ones.

The $\{Z_2^{(1)}, Z_2^{(2)}, Z_2^{(1)} \times Z_2^{(2)}\}$ structure can not only be realized through so-called non-freely acting projections (i.e. pure twists) but also by letting one or two of these projections to act freely. Let us indicate the structure of the "pure-twist" orbifold as (t,t,t). It is easy to see that, by an appropriate choice of shifts to be associated to the twists, it is also possible to realize the structures (s,t,t), (s,s,t) and (s,s,s), where s and t respectively indicate the nature of the projection ($s =$ all states shifted; $t =$ pure twist) on the first, second and third complex orbifold plane. Indeed, all these constructions belong to the string phase space, and contribute to the overall appearance of the string realization of the scenario described

[9] As we will discuss, the leptons show up as singlets inside quark multiplets.

by 1.1. The difference between these configurations is that in the (t,t,t) realization we have a replication of the matter states into three families, in the (s,t,t) realization we have just two families, in the (s,s,t) one family, whereas in the (s,s,s) there is no matter at all (for a general discussion and classification of these cases, see Ref. [17]). The existence of all these realizations of the $Z_2 \times Z_2 \times Z_2$ orbifold plays a key role for the mass differentiation between matter families.

2.1.2 Origin of four dimensional space-time

The product (2.7) represents the maximal number of independent twists the theory can accommodate: a further twist would in fact superpose to the previous ones, and restore in some twisted sector the projected states. Therefore, further projections are allowed, but no further twists of coordinates. These twists allow us to distinguish between "space-time" and "internal" coordinates. While the first ones (the non-twisted) are free to expand, the twisted ones are "frozen". The reason is that the graviton, and as we will see the photon, propagate along the non-twisted coordinates, and therefore expand the universe by stretching its horizon, allowing us to perceive these coordinates as our "space-time". We get therefore "a posteriori" the justification of our choice to analyze sectors and moduli from the point of view of a compactification to four dimensions.

2.1.3 In how many dimensions does non-perturbative String Theory live?

Besides the above mentioned twists/shifts, the only way to further minimize symmetry is to apply further shifts along the non-twisted coordinates. How many are they? From the type II point of view, there are no further, un-twisted coordinates. But we know that they are there, "hidden" as longitudinal coordinates eaten in the light-cone gauge and in the coupling of the theory. Some of these coordinates appear on the heterotic/type I side as *two* transverse coordinates. If we count the total number of twisted coordinates by collecting the information coming from intersecting dual constructions, and the coordinates which are "hidden" in a certain construction and are explicitly realized in a dual construction, we get the impression that the underlying theory possesses 12 coordinates. For instance, on the heterotic side we have a four-dimensional space-time plus six internal, twisted coordinates, and a coupling. On the type II side we see eight twisted coordinates. We would therefore conclude that the two additional twisted coordinates correspond to the coupling of the heterotic dual. On the other hand, no supersymmetric 12-dimensional vacuum seems to exist, at least not in a flat space: the maximal dimension with these properties is 11. This seems therefore to be the number of dimensions in which non-perturbative string theory is natively defined. Let's have a better look at the properties of supersymmetry. As is known, the supersymmetry algebra closes on the momentum operator. When applied to the vacuum, we have:

$$\left\{ Q, \bar{Q} \right\} \approx 2\, M \, . \tag{2.9}$$

From a dimensional point of view, a mass can be viewed as the inverse of a length, so that we can also write:

$$< \{Q, \bar{Q}\} > \cong \frac{1}{R} \,. \tag{2.10}$$

The supersymmetry algebra suggests that the mass on the right hand side of 2.9, in all respects an order parameter for the supersymmetry breaking, could be interpreted as the inverse of the length of a coordinate of the theory. This coordinate refers to an extra internal dimension, or, perhaps more appropriately, to a curvature, i.e. a function collecting the contribution of several coordinates, perturbative as well as non-perturbative. We can therefore view the supersymmetric phase as the limit $R \to \infty$ of a theory with generically broken supersymmetry. This decompactification is only possible if the coordinate R is not twisted. Precisely the fact that, in the breaking of $\mathcal{N}_4 = 2$ supersymmetry to $\mathcal{N}_4 = 1$, the dilaton and the other "coupling" fields get twisted, is a signal that a non-vanishing curvature of the string space has been generated. As we discussed in section 2.1.1, this means that, even in the case of infinite volume, we are in a situation of non-compact orbifold. In the orbifold language, this is implemented by the fact that, whenever the coupling field is "explicitated" by going to a dual construction, the corresponding perturbative geometric field appears as a volume of a two-dimensional space. This phenomenon can be observed for reduced supersymmetry (for maximal supersymmetry, there is just the type II string construction). Consider for instance the eleventh coordinate of M-theory, that should correspond to the dilaton of the heterotic string. In the type II orbifold constructions (K3 orbifold compactifications), the heterotic coupling corresponds to a two-torus volume. Considering that this two-dimensional space corresponds, from the heterotic point of view, to "extra-coordinates", one would say that, in order to realize all these degrees of freedom, the full underlying theory should be (at least) twelve-dimensional. However, this is only an artifact of the linearization implied by the orbifold construction, and it means that the simple compactification on a circle is not enough, we need an additional coordinate in order to parametrize a curved space in terms of flat coordinates. From the type II dual we learn that supersymmetry is not restored by a simple decompactification: the string space is twisted [10]. Flatness of the string space is broken by a "twist" of coordinates that fixes them to the Planck scale. As a consequence, the supersymmetric partners of the low-energy states are boosted above the Planck scale. In a situation of supersymmetry restoration, they should come down to the same mass as the visible world, and space should become "flat". However, this is only possible when the twist is "unfrozen" and we can take a decompactification limit, such as for instance the M-theory limit. Otherwise, at the decompactification limit the space becomes only locally flat (non-compact orbifold). Let's collect the informations so far obtained:

[10] In some type II/heterotic duality identifications, the heterotic coupling is said to correspond to un-twisted coordinates of the type II string. This however does not change the terms of the problem: in the artifacts of the flattening implied by the orbifold constructions, part of the curvature may be "displaced", referred to some or some other coordinates. This "rigid" distribution of the twists, basically dictated by the need of recovering a description in terms of supergravity fields referring to the same space-time dimensionality for both the dual constructions, may induce to misleadingly conclusions. The intrinsic twisted nature of the space has to be considered by looking at the string space in its whole (for more details and discussion, see for instance Ref. [17]).

1. As soon as the string space is sufficiently twisted, supersymmetry is broken.
2. Equations 2.9 and 2.10 suggest in this case a non-vanishing curvature of space.
3. In the class of orbifolds, the phenomenon of curving the string space can only be partially and indirectly seen, through the comparison of dual constructions.
4. These constructions are built on a (perturbatively) flat, supersymmetric background: they provide therefore "linearizations" of the string space.
5. The maximal dimension of a supersymmetric theory on a flat background is 11.

All this suggests that, when supersymmetry is broken, we are in the presence of an eleven-dimensional *curved* background. Any, forcedly perturbative, explicit orbifold realization requires for its construction a linearization of the background. Since a 11-dimensional curved space can be embedded in a 12-dimensional flat space, we have the impression of an underlying 12-dimensional theory. However, this is only an artifact; in fact, we never see all these 12 flat coordinates at once: we infer their existence only by putting together all the pieces we can explicitly see. But this turns out to be misleading: the linearization is an artifact.

The 12 dimensional background is only fictitious, we need it only in order to describe the theory in terms of flat coordinates. At the perturbative string level, of these coordinates we see only a maximum of 10.

As a matter of fact, we are however in the presence of a maximum of seven "twisted" coordinates, i.e. coordinates along which the degrees of freedom don't propagate, and four un-twisted ones, along which the degrees of freedom can propagate. By comparison of dual string vacua, we can see that there is room to accommodate two more "perturbative" Z_2 shifts: through the heterotic and/or type I realization in the light-cone gauge we can explicitly see two more transverse coordinates which are non-twisted, along which we can accommodate two further independent shifts, plus two longitudinal ones, along which no shift can act.

2.1.4 Shifting the space-time

Let's count the number of degrees of freedom of the matter states. We have three families, that for the moment are absolutely identical: each one contains 4 (massless) chiral fermions with an "internal" multiplicity 4. The number of matter degrees of freedom is therefore the right one in order to build up three families of *massive* doublets of quarks (with multiplicity 3 out of the 4 of the internal symmetry) and leptons (with multiplicity 1 out of the 4 of the internal symmetry). The differentiated spectrum of massive quarks and leptons originates out of this bunch of equivalent massless degrees of freedom as a consequence of being the universe the superposition of several configurations obtained by further shifting the string space along the non-twisted, transverse space coordinates. Shifts applied to the "internal" coordinates reduce the number of states through mass lifts that, owing to the fact that the coordinates are also twisted, remain for ever fixed (in this specific case, at the Planck scale). Also the shifts acting on the space-time coordinates reduce further the symmetry group.

However, the matter states "projected out" by these shifts are not thrown out from the spectrum of the low energy theory: they acquire a mass difference inversely related to the scale of space-time.

Let us consider the possible actions of the two further shifts. The resulting physical configuration will be given by the superposition of all of them, with a "weight" that we will determine in detail in sections 4 and 5. First of all, we notice that with one shift we can make all the three matter sectors, namely what remains of the twisted $Z_2^{(1)}$, $Z_2^{(2)}$ and $Z_2^{(1)} \times Z_2^{(2)}$, freely acting (for instance by pairing $Z_2^{(1)}$ to a shift on the first space coordinate, $Z_2^{(2)}$ with a shift on the second one, so that $Z_2^{(1)} \times Z_2^{(2)}$ is associated with a shift along both of them). This can be indirectly observed in the type II realization of the orbifold, where one can explicitly observe the Cartan subgroup of the gauge group. From its rank, one sees how many matter states, which transform in the fundamental representation of the group, do survive. This is particularly easy to observe in the $\mathcal{N}_{(4)} = 2$ orbifold constructions of the orbifold. An explicit realization of the further step, $\mathcal{N} = 0$, is given in Appendix B, where the supersymmetry breaking is made explicit by trading the space coordinates for the non-perturbative plane of the coupling. The full breaking of supersymmetry and the reduction of states through shifts along the space coordinates cannot therefore be explicitly observed in the same dual construction. In order to follow the process of reduction/shifting of states, one can imagine to apply first the shifts along the space coordinates, and then reduce supersymmetry to $\mathcal{N} = 0$. The result of this operation is that we make all the matter states massive. This also means that each fourth-plet of massless chiral fermions is turned into a doublet of massive states. There are no gauge bosons of the weakly coupled sector at all: all of them have been shifted (remember however that each one of these sectors remains charged under an internal **4**, which is at the strong coupling). On the type II string realization gauge charges are always invisible, but from the way matter states and corresponding gauge bosons are realized on the heterotic constructions we learn that they originate from dual sectors: if the matter states are lifted by a shift on the momenta of the lattice of the untwisted coordinates, the corresponding gauge bosons are lifted in a T-dual way. A sub-Planckian mass for the matter states is associated to an over-Planckian mass of the corresponding lifted gauge bosons. In this case, the lattice under consideration is the one of the momenta (and windings) of the transverse space coordinates, that we consider compactified.

After this shift operation, there is room for a second, independent shift, of the "rank reducing" type like those considered in the section 2.1.1. In this way, half of the states of a twisted sector, i.e. precisely one massive fermion, already lifted to a non-vanishing mass by the first shift, gets further lifted. Depending on whether we associate this second shift to one or two Z_2 twist operations, we can either i) further lift half the states of one twisted sector, lift done half the states of a second twisted sector, leave unchanged the states of a second one while in the third one the shifts superpose and rearrange leaving the spectrum unchanged, or ii) leave unchanged the states of two twisted sectors and lift down half of the states of the third sector. Owing to the presence of sectors in which all four chiral fermions are shifted in the same way, there are configurations in which survive also the gauge bosons of a $SU(2)$ symmetry which rotates chiral fermions. Another way to apply the two shifts is to avoid a full freely-acting operation in the first shift, letting each one of the two shifts to act independently as "rank reducing" by lifting just half of the states. Also in this case, the product of the two operations acts differently on the three orbifold twisted sectors, because on the product of the two independent orbifold twists the shifts superpose and neutralize their effect against each other. Interesting is however that the first shift produces configurations in which of the four massless chiral fermions of a twisted sector one obtains one massive (non-chiral) state and two chiral massless transforming under an $SU(2)$ symmetry. The second shift operation breaks then also this symmetry. On the other hand, the two massless chiral fermions appearing in some configuration can also be arranged in order to be seen as the left and right moving part of just one state, that eventually, owing to the superposition with the other configurations, is going to acquire a mass. In this way, we have configurations in which there is one $U(1)$ boson which couples non-chirally.

All in all, when all these configurations are superposed, we have only doublets of massive states, which transform under a "softly broken" chiral $SU(2)$ symmetry. By this we mean that each of the massless fourth-plets of the former twisted sectors are reduced to pairs of massive fermions, which do not really possess an $SU(2)$ symmetry, because they are slightly differentiated in mass. Nevertheless, a chiral part, that by convention we may choose to be the left-moving part, of each massive state, in some configurations transforms under an $SU(2)$ symmetry, which is broken by the superposition of configurations leading to a mass differentiation of the states, while there is a certain amount of non-chiral coupling to $U(1)$. This is the mechanism of soft breaking of the $SU(2)$ weak symmetry in our scenario.

2.1.5 The photon and the $SU(3)$ of QCD

The $Z_2^{(1)} \times Z_2^{(2)} \times Z_2^{(3)}$ orbifold is symmetric under the exchange of each projection with each other. As a consequence, the superposition of configurations which breaks the symmetry in the weak sectors analogously breaks also the strong sector: also the internal $\mathbf{4}$ of each bi-charged fourth-plet gets broken, this time however forcedly into singlets, $\mathbf{1}_1 \oplus \mathbf{1}_2$, with a slight mass differentiation between the states trasforming under the $\mathbf{1}_1$ and those transforming under $\mathbf{1}_2$. Since the gauge sector is at the strong coupling, the *interpretation* we must give to this breaking is that the initial $\mathbf{4}$, corresponding to $U(4)$, has been broken into $\mathbf{1} \oplus \mathbf{3}$, i.e. the gauge group to $U(1) \times (U(1) \times SU(3))$, the $\mathbf{1}_1$ being a trivial singlet of $SU(3)$ corresponding to a state charged only under $U(1)$, the $\mathbf{1}_2$ being instead a singlet of $SU(3)$ made out of three charged states. In practice, each fourth-plet of states charged under a chiral $SU(2)$ breaks into a singlet and a triplet of $SU(3)$. Furthermore, all these states are charged under $U(1)$, that, having already taken out a phase from the gauge group of the weak sector, we attribute to the strong sector. Each fourth-plet gives therefore rise to what we may identify as one lepton and three quarks. Coming from the breaking of an $SU(4)$ symmetry, the $U(1)$ factor is traceless. This means that it acts by transforming with opposite phase states charged under $SU(3)$ and uncharged ones:

$$
\begin{aligned}
U(1)\,\varphi &= \mathrm{e}^{\mathrm{i}\beta}\,\varphi\,, \\
U(1)\,\varphi_a &= \mathrm{e}^{-\mathrm{i}\,\beta/3}\,\varphi_a\,, \qquad a \in \mathbf{3}\,\text{of}\,SU(3)\,.
\end{aligned}
\tag{2.11}
$$

Here φ indicates a full chiral fourth-plet of the weak sector. These states, as we have just seen, arrange into massive doublets of a broken weakly coupled $SU(2)$, that we identify with the symmetry group of the weak interactions. The condition on the trace of $U(1)$ holds for $SU(2)$ doublets, but tells nothing about the charge assignments among the states of each $SU(2)$ pair. This indication comes from a further condition, namely the fact that the *strength* of the $U(1)$ interaction is in this context by definition related to the weight this interaction has in the phase space of all the configurations. Since quarks occur three times more than leptons (remember that each fourth-plet, or equivalently each $SU(2)$ doublet pair, bears an internal multiplicity $\mathbf{4} = \mathbf{1}_{\text{leptons}} + \mathbf{3}_{\text{quarks}}$), we obtain the following condition on the charge Q:

$$\sum_{\text{quarks}} |Q(U(1))| \;=\; 3 \sum_{\text{leptons}} |Q(U(1))| \,. \tag{2.12}$$

Besides this, we have also the condition 2.11 on the trace that in terms of the charge can be written as:

$$\sum_{\text{leptons}} Q(U(1)) \;=\; - \sum_{\text{quarks}} Q(U(1)) \,. \tag{2.13}$$

The fact that these conditions hold separately for each of the three matter families implies that in each family there must be one state with $Q(U(1)) = 0$. This must necessarily be identified with the lightest particle of each family. If we call the leptons of the fourth-plet in the usual way neutrino and electron, and the quarks down and up quak, and set by convention $Q_e = -1$, from 2.11, 2.12 and 2.13 we derive the charge assignments $Q_\nu = 0$, $Q_u = 2/3$, $Q_d = -1/3$. The $U(1)$ gauge group has all the characteristics of $U(1)_\gamma$, the group of electromagnetism. The corresponding vector field is the photon, and the neutrino, being the less interacting particle, must be identified with the lightest of the fourth-plet.

The spectrum does not contain the degrees of freedom of a possible Higgs boson. On the other hand, here there is no need of such a field, because masses are generated with a pure stringy mechanism, and are basically related to the compactness of the whole space. As remarked in [44], the Higgs boson of ordinary field theory can in some way be thought as the parametrization of a boundary term through a field propagating in the bulk of space [11] (in section 7.1 we will comment about the 125 GeV resonance detected at LHC ([45]), and usually seen as a signal of the Higgs boson).

[11] It is legitimate to ask what is the mass scale of the gauge bosons of the "missing" $SU(2)$, the would-be $SU(2)_{\text{R}}$ of the original weak fourth-plet, $\mathbf{4} = \mathbf{2}_{SU(2)_{\text{L}}} + \mathbf{2}_{SU(2)_{\text{R}}}$. Namely, asking whether there is a scale at which we should expect to observe an enhancement of symmetry. The answer is: there is no such a scale. The reason is that the scale of these bosons is simply T-dual, with respect to the Planck scale, to that of the masses of particles. Let's consider this shift as seen from the heterotic side. On the heterotic vacuum, matter states originate from the twisted sector, while the gauge bosons (the visible gauge group, the one involved in this operation) originate from the currents, in the untwisted sector of the theory. Similarly, on the type I side, gauge bosons and the charged states we are considering originate from D-branes sectors derived respectively from the untwisted, and the twisted orbifold sectors of the starting type II theory [12]. It is therefore clear that a shift on the string lattice lifts the masses of gauge bosons and those of matter states in a T-dual way. Since the scale of particle masses is below the Planck scale, the mass of these bosons is above the Planck scale; at such a scale, we are not anymore allowed to speak of "gauge bosons" or, in general, fields, in the way we normally intend them.

2.2 *The order of the symmetry breaking*

What we have analyzed are the various configurations of the maximal Z_2 orbifold one may construct, namely a product of $Z_2^{(1)} \times Z_2^{(2)} \times Z_2^{(3)}$ twists, which may act as pure twists or as partially or totally freely-acting. The difference between the various configurations depend on the different ways the shifts can be taken. The result is a set of configurations with almost the same weight in the phase space, because obtained with the same amount of projections. As a consequence, their superposition produces a spectrum in which the states are distinguished by mass differences of the order of the inverse of the length of space, or equivalently the age of the universe. Being obtained in a logarithmic representation (see Ref. [2]), once pulled back to the physical representation the masses are expected to be of the type:

$$m_i \approx \frac{1}{\mathcal{T}^{p_i}}, \tag{2.14}$$

for a series of exponents $0 < p_i < 1$. String configurations with a lower amount of projections, such as for instance those with a spectrum of full massless fourth-plets, or with a lower number of matter families, etc..., do not contribute to what we call the spectrum of the theory, i.e. the number and type of fundamental particles and fields, but to what we identified as the bunch of "quantum corrections". The reason is that one Z_2 projection less means a factor 1/2 in the weight of a string configuration in the logarithmic space, i.e. a square-root factor of the weight of the configuration in the true phase space of 1.1. As discussed in [1], the configurations of highest entropy have a weight that goes like $\mathrm{e}^{N^2} \sim \mathrm{e}^{\mathcal{T}^2}$. A square root factor means a suppression of order $\mathrm{e}^{-N} \sim \mathrm{e}^{-\mathcal{T}}$.

2.2.1 *Breaking the symmetry between the three matter families*

In the (t, t, t) realization of the orbifold all the three matter families acquire an identical mass. The same is true also for the (s, t, t) and (s, s, t) realization, with the only difference that the number of matter families is two and one respectively: in all these realizations the shift along the space coordinates produces the same mass spectrum, replicated along the families. However, in the string phase space the configuration with one matter family is realized three times more frequently than the one with three families, and the configuration with two families two times more frequently. If we call first family the one which is present in all the three realizations, second family the one which is present in two of them, and thrid family the one which is present in all the three, this means that the third family weights three times more, and the second family two times more, than the first one. Namely, on the string realization, that we recall is a logarithmic realization of the physical configuration, the volumes occupied in the phase space stay in ratio 1:2:3. Since the symmetry group corresponding to the rotation of families is only a part of the whole symmetry group of the configurations, this does not imply that, once pulled back to the physical (= exponential) representation masses stay in relation $m_{(3)} = m_{(2)}^{3/2} = m_{(1)}^3$, but that if the ratio $m_{(2)}/m_{(1)}$ is a certain (possibly time-dependent) factor $X_{2/1}$, then the ratio $m_{(3)}/m_1$ is given by $X_{2/1}^2$, i.e., the masses of the third and second family have the same logarithmic distance than the masses of the second and first one. Notice that shifts along the space coordinates break the Lorentz symmetry. Therefore, the superposition of differently shifted configurations, in particular of those of highest entropy, not only implies the breaking of parities, but also the breaking of the symmetry of space under rotations. This occurs at the same time as masses are produced: the amount of breaking of space rotations produced is of the same order of the particle masses.

2.3 The fate of the magnetic monopoles

Under the conditions of the scenario we are discussing, namely of a universe "enclosed" within a finite, compact space, also the issue of the existence of magnetic monopoles changes dramatically. Magnetic monopoles can be of two kinds: the "classical" ones, namely those associated to a non-vanishing "bulk" magnetic charge that parallels the electric charge in a symmetric version of the Maxwell's equations, and the topological ones. In our scenario there are neither classical nor topological monopoles. The existence of classical monopoles would be possible only in the absence of an electro-magnetic vector potential, what we have called the "photon" A_μ; their existence has therefore been ruled out as soon as we have discussed the existence and the masslessness of this field. The first idea about the existence of magnetic monopoles in the classical sense (i.e. non-topological) originated by a request of symmetry: were not for the absence of magnetic charges, the Maxwell equations would be completely symmetric in the electric and magnetic field. However, the symmetry of these equations, preserved in empty space, is precisely spoiled by the presence of matter states that are also electrically charged. In our scenario, the description of the universe is "on-shell" and the presence of matter comes out as "built-in": it cannot be disentangled from the existence of space itself. In this scenario there are no topological monopoles either. As all vector fields are twisted (i.e. massive at the Planck scale or above it) with the only exception of the photon A_μ, propagating in the four-dimensional space time, and as this space-time dimensionality is electro-magnetically self-dual, the only possible topological monopoles would be those of the four-dimensional space coupled to the same photon field A_μ, namely, configurations à la t'Hooft and Polyakov or similar [13]. However, any such topological configuration is characterised by its being living in an infinitely-extended space: only in this way it is in fact possible to make compatible the existence of a p-form working as a "potential" $A_{(p)}$, defined as an analytic function in every point of the space, with the presence of a non-trivial magnetic flux. As is well known, the magnetic flux through a surface can be computed as a loop integral of the vector potential. In the case of a surface enclosing a finite volume, the total flux is the sum of the loop integral circulated in both the opposite directions, so that it always trivially vanishes. However, things are different if the field has a non-trivial behaviour at infinity. At infinity we need just the circulation in one sense, because there is no "outside" from which field lines can "re-enter" in the space: if there is a non-vanishing circulation, there is a non-vanishing magnetic flux, and therefore also a non-vanishing magnetic charge. This however also means that, provided it exists, such a magnetic monopole is a highly non-localised object, with a magnetic field/vector potential such that the magnetic flux vanishes through any compact finite closed surface

[13] for a review and references, see for instance [46, 47].

[14]. As a consequence, also the magnetic charge density vanishes point-wise at any place in the "bulk". Therefore, in our setup, where space is compact, these monopoles cannot exist. Moreover, in our case we don't have a Higgs mechanism either, and, since the surface at infinity does not belong to any configuration of space-time, there is no smooth limit with a true restoration of the conditions at infinity allowing the existence of non-trivial topologies and homotopy groups. Light states with topological magnetic charges do not exist at all, not even approximately as the time becomes very large [15].

[14]Notice that the situation around the zero-dimensional point is equivalent to the one around the surface at infinity: if on one side the Dirac string can be considered as somehow the "dual" picture of the surface at infinity of the t'Hooft and Polyakov construction, in our scenario both infinity and the dimensionless point are excluded. Differential geometry and gauge theory are here only approximations.

[15]The situation is similar to the case of the volume of the group of translations and its identification with the regularized volume of space in the usual normalization of operators and amplitudes, completely absent in our scenario, something that leads to a different interpretation of string amplitudes as global quantities instead of densities, cfr. Ref [44].

3 The geometry of the universe

As discussed in [2], the absence in our theoretical framework of symmetry under space-time translations implies a different normalization of string amplitudes, which must be now normalized in such a way that densities scale like the inverse of the Jacobian of the transformation between string world-sheet and target space coordinates. An amplitude which in the light-cone gauge is of order one, like the vacuum energy in the non-supersymmetric orbifolds considered in the previous section, in which supersymmetry is broken at the unit scale (identified with the Planck scale), gives therefore an energy density which scales as:

$$\rho(E) \sim \frac{1}{\mathcal{T}^2} . \tag{3.1}$$

In order to get the value of a global quantity, like the entropy, we must instead multiply the string amplitude by the Jacobian factor, obtaining the scaling:

$$S \propto \mathcal{T}^2 . \tag{3.2}$$

The total energy at a certain time $\mathcal{T}$ of the history of the universe, given by the integral of the energy density over the space volume of the universe at time $\mathcal{T}$, scales then as:

$$\mathrm{E}(\mathcal{T}) \sim \int_{\mathcal{T}} d^3 \frac{1}{\mathcal{T}^2} \approx \mathcal{T} . \tag{3.3}$$

In the string representation we recover therefore the values we computed in the ground description of this scenario.

3.1 The solution of the FRW equations

The density 3.1 collects both the pure geometric, i.e. cosmological, and the matter/radiation contribution to the energy density. These terms are separately of the same order. The reason is that the set of most entropic string vacua inherits what remains of the symmetry under exchange of three sectors of the theory at the $\mathcal{N}_4 = 2$ level, the $S - T - U$ symmetry of the orbifold construction, which can be seen to exchange the roles of gravity, matter and radiation by exchanging the sectors giving rise to the corresponding fields. In the further steps of symmetry breaking this symmetry is broken by terms of order $\mathcal{O}(1/\mathcal{T}^p)$ in the string partition function. The energy densities get therefore distinguished by higher order terms: $\rho \sim \frac{1}{\mathcal{T}^2} \longrightarrow \frac{1}{\mathcal{T}^2} (1 + \mathcal{O}(1/\mathcal{T}^p))$.

Let us now investigate the geometry of the expansion of the universe. As the universe evolves, the energy density and the curvature of space-time decrease toward a flat limit, and the dominant configuration tends to a "classical" description. At large $\mathcal{T}$ it is therefore reasonable to suppose that this configuration admits a description in terms of Robertson-Walker metric, i.e. a classical metric of the type:

$$ds^2 = dt^2 - R^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2) \right] , \tag{3.4}$$

where for us $t \equiv \mathcal{T}$, and $r \leq 1$. The metric should correspond to a closed universe, $k = 1$. Under the assumption of perfect fluid for the energy-momentum tensor, the Einstein's equations lead to:

$$\left(\frac{\dot{R}}{R} \right)^2 = - \frac{k}{R^2} + \left\{ \frac{8\pi \, G_{\rm N} \, \rho}{3} + \frac{\Lambda}{3} \right\} , \tag{3.5}$$

where we have collected within brackets the contribution of the stress-energy tensor and of the cosmological term. Inserting the "Ansatz" $R = \mathcal{T}$ we obtain:

$$\left(\frac{\dot{R}}{R} \right)^2 = - \frac{(k=1)}{R^2} + \left\{ \frac{\sim 2}{R^2} \right\} \sim \frac{1}{R^2} \, , \tag{3.6}$$

that we can write as:

$$\left(\frac{\dot{R}}{R} \right)^2 = \frac{\kappa^2}{R^2} \, , \tag{3.7}$$

for some coefficient κ. The equation is solved by $R = \kappa \, t$, consistently with our Ansatz. This confirms that the dominant configuration corresponds to a spherical Robertson-Walker metric, describing a universe bounded by a horizon expanding at a fixed ratio to the speed of light.

The comparison of our results with astronomical data contains however a possible weak point. Experimental data are given as a result of a process of interpretation of certain measurements, for instance through a series of interpolations of parameters. All this is consistently done within a well defined theoretical framework. Usually, one takes a "conservative" attitude and lets the interpolations run in a class of models. However, this is always done within a finite class of models. In principle, we are not allowed to compare theoretical predictions with numbers obtained through the elaboration of measurements in a different theoretical framework: in general, this doesn't make any sense. However, in the present case this comparison is not meaningless, and this not on the base of theoretical grounds: the reason is that, for what concerns the time dependence of cosmic parameters and energy densities, the solution we are proposing does not behave, at present time, much differently from the "classical" cosmological models usually considered in the theoretical extrapolations from the experimental measurements. The rate of variation of energy density is in fact: $\dot{\rho} \sim \partial(1/R^2)/\partial \mathcal{T} = 1/\mathcal{T}^3 = 1/R^3$. The values of the three kinds of densities can therefore be approximated by a constant within a wide range of time. For instance, as long as the accuracy of measurements does not go beyond the order of magnitude, these densities can be assumed to be constant within a range of several billions of years. For the purpose of testing the statements and conclusions of the present analysis, the use of the known experimental data about the cosmological constant, derived within the framework of a Robertson-Walker universe with constant densities, is therefore justified.

A universe evolving according to eq. 3.6 is not accelerated: $\dot{R} = 1$ and $\ddot{R} = 0$. Owing to the existence of an effective Robertson-Walker description, the red-shift can be computed as usual. We have:

$$1 + z = \frac{\nu_1}{\nu_2} = \frac{R_2}{R_1} = \frac{\mathcal{T}_2}{\mathcal{T}_1}, \tag{3.8}$$

where ν_1 is the frequency of the emitted light, ν_2 the frequency which is observed, and R_1, R_2 are respectively the scale factor for the emitter and the observer. $R = \mathcal{T}$ is precisely the statement that the expansion is not accelerated. Expression 3.8 however accounts for just the "bare" red-shift, namely the part due to the expansion of the universe: it does not account for the corrections coming from the time dependence of masses, that we will discuss in section 4.3.7. Usually, this effect is not taken into account, because in the standard scenarios masses are assumed to be constant. In our scenario they depend instead on the age of the universe. A change in the values of masses reflects in a change of the atomic energy levels, and therefore in a change of the emitted frequencies. We will see that, once the observed frequencies in expression 3.8 are corrected to include also the change in the scale of energies, the scaling of the emitted to observed frequency ratio is not anymore proportional to the ratio of the corresponding ages of the universe. Since the conclusions about the rate of expansion are precisely derived by comparing red-shift data of objects located at a certain space-time distance from each other, this explains why the expansion *appears to be accelerated*.

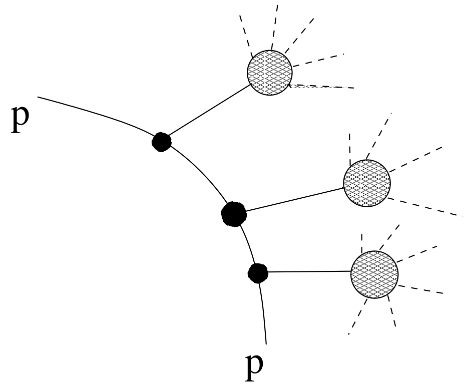
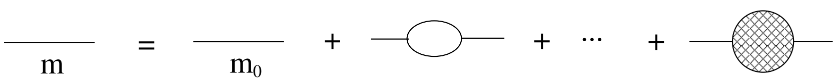


Figure 1: The more and stronger are the interactions of the particle p, the higher its entropy.

4 Masses and couplings

4.1 The mass of a particle

We consider now the masses of the elementary particles. In our approach, a particle is a certain localized amount of energy that rests or moves in space (indeed, the difference between a massive particle and a massless one relies precisely in the fact that the massive particle can stay at rest, whereas the massless one can not). When this scenario is translated into the string representation, masses arise as ground momenta associated to the states of the string spectrum. Since through 1.2 the string scenario is a representation of the combinatorial one, even in the string space a mass is related to the weight of a certain state in the phase space. In the ordinary perturbative approach to field theory (no matter whether it is string-inspired, string-derived, or not) masses, after they have been introduced via some mechanism (Higgs mechanism), are attributes which in general receive corrections at various perturbative orders. The corrections appear as the sum of a series of insertions in the free propagator:



(4.1)

Mass and volume in the phase space are related by the fact that the more are the decay channels of a particle, the larger is its entropy, and also the correction to the mass, because higher is the number of virtual lines (Feynman diagrams) contributing to the mass renormalization (all this is illustrated in figure 1 of page 28). Heavier particles possess a huger decay phase space: quarks are heavier than leptons, and among leptons neutrinos are the lightest particles. Inside each family of particles, the heavier (for instance the top as compared to the bottom of an $SU(2)$ doublet) has the larger absolute value of the electroweak charge. In each family, the lightest particle is the one which has less interactions, or less charge (and therefore a lower interaction probability). For instance, $|Q_\nu| < |Q_e|$, $|Q_b = -1/3| < |Q_t = +2/3|$, and quarks, that feel also the $SU(3)$ interactions, are heavier than leptons [16]. Along this line, we can view the lightest particle as the end-point of a chain of projections that reduce the symmetries of the internal space. Heavier particles are therefore those which "occupy" a larger space; they correspond to a larger internal symmetry than lighter particles. Lighter particles correspond to sub-volumes, sub-spaces of those of the heavier particles: the phase space of lighter particles is contained in the phase space of heavier ones. To figure out this point, think for instance at the case of a heavy particle that decays into lighter ones: the physics of these latter is "contained", in the sense that it is produced, derived, by the physics of the heavier one. In terms of combinatorials of distribution of energies, this simply means that the ways of distributing an amount of energy E along space contain the ways we can distribute an amount $E' < E$.

4.2 The couplings

In this scenario, also couplings are related to weights in the phase space; ratios of masses and couplings are related. The ratios of masses correspond to ratios of symmetries, i.e. to cosets. When we pass from a particle to another, lighter one, we reduce the symmetry of the subset of the configuration of the universe (i.e. the subgroup of the whole symmetry group representing the configuration of the universe) which represents the particle under consideration. We say we break a certain group to a subgroup. The inverse of the volume of the coset is what we call "coupling". Indeed, in the case we start from a particle and follow a decay process leading to some products (particles and fields with a certain energy), the ratio of the volume of the initial particle to the volume of its decay products gives the coupling of the interaction. Interactions are therefore in a natural way related to groups and symmetries. We will see that this point of view includes both the symmetries which, in the representation in terms of fields living on the continuum, correspond to ordinary gauge symmetries, and the symmetries that in such a representation are going to be "hardly broken", i.e. appear as rigid symmetries. This second type does not give rise to interactions propagating through gauge bosons, but describes for instance the leading contribution to "transitions" such as the passage from a family of quarks or leptons to another one. In this interpretation, they are viewed as an appropriate type of rotations in the phase space. In both cases, these processes, and their relative couplings, follow the rules of composite probabilities typical of the weights in the phase space. A composite transition/decay process: $A \to B \to C$, corresponds to a rotation with an element of the group $\mathbf{G}_{(AC)}$, given by a product $\mathbf{G}_{(AC)} = \mathbf{G}_{(AB)} \times \mathbf{G}_{(BC)}$. This transition corresponds therefore to: 1) first a rotation with an element of the group $\mathbf{G}_{(AB)}$ and then: 2) a rotation with an element of $\mathbf{G}_{(BC)}$. Therefore, we expect the probability

[16] The first quark family makes an apparent exception: the down quark is heavier, although less charged, than the up quark. This issue will be discussed in detail in section 5.2.

of the decay $A \to C$ to be the product of the decay probability of $A \to B$ and of $B \to C$:

$$P(A \to C) \,=\, P(A \to B) \times P(B \to C)\,. \tag{4.2}$$

The effective coupling determines the probability amplitude:

$$P(A \to B) \,\sim\, \alpha_{(AB)}\,, \tag{4.3}$$

where, as usual, $\alpha_{(AB)} \equiv g^2_{(AB)}/4\pi$. The effective coupling for the transition from A to C is given by the product of the effective couplings of the single steps:

$$\alpha_{(AC)} \,\propto\, \alpha_{(AB)} \times \alpha_{(BC)}\,. \tag{4.4}$$

The square coupling of the group $\mathbf{G} \,=\, \mathbf{G_1} \otimes \mathbf{G_2} \otimes \ldots \otimes \mathbf{G_n}$ is therefore:

$$\alpha_{\mathbf{G}} \,=\, \alpha_{\mathbf{G_1}} \times \alpha_{\mathbf{G_2}} \times \cdots \times \alpha_{\mathbf{G_n}}\,. \tag{4.5}$$

In the usual renormalization group approach one works in the algebra $\mathcal{G}$ of the group $\mathbf{G}$. If $\mathbf{G} \,=\, \mathbf{G_1} \otimes \mathbf{G_2} \otimes \ldots \otimes \mathbf{G_n}$, the algebra decomposes as $\mathcal{G} \,=\, \mathcal{G}_\mathbf{1} \oplus \mathcal{G}_\mathbf{2} \oplus \ldots \oplus \mathcal{G}_\mathbf{n}$, and the inverse of the effective coupling seems to renormalize additively. For instance, the one-loop beta-function of $SU(N)$ with gauge bosons in the adjoint and (massless) matter states in the fundamental representation is one half of the beta-function of $SU(2N)$ with an analogous content of matter and gauge states. From the point of view of our approach, what seems to behave additively is just the logarithmic derivative of the inverse of the coupling.

4.3 Entropy and mass

We will now see in detail how the ratios of masses are in relation to the ratios of the volumes of these groups, in turn related to the ratios of the strengths of the corresponding couplings. According to our previous discussion, the strength $\alpha(G)$ is by definition proportional to the volume of the group, $||G||$ (not to be confused with the volume of the Lie algebra $||\mathfrak{g}||$), and we can write:

$$\frac{\alpha(G_i)}{\alpha(G_j)} \,=\, \frac{||G_i||}{||G_j||}\,. \tag{4.6}$$

On the other hand, also masses are related to volumes of symmetries, so that we can write a similar expression:

$$\frac{m_k}{m_\ell} \,=\, \frac{||G_k||}{||G_\ell||}\,. \tag{4.7}$$

By comparison of these two expressions we obtain:

$$\frac{m_k}{m_\ell} \,=\, \frac{\alpha(G_k)}{\alpha(G_\ell)}\,, \tag{4.8}$$

This expression can also be written as:

$$\frac{m_i}{m_j} \,=\, \alpha(G_{ij}) \,=\, ||G_{ij}||\,, \tag{4.9}$$

where G_{ij} is a coset. In the logarithmic picture the couplings read:

$$\left.\frac{1}{\alpha_i}\right|_{\log} \;=\; \frac{1}{\alpha_0} \,+\, \beta_i \ln \mu \,. \tag{4.10}$$

Ratios become differences, and we can write:

$$\frac{\alpha_i}{\alpha_j} \;\to\; \left.\frac{1}{\alpha_i}\right|_{\log} - \left.\frac{1}{\alpha_j}\right|_{\log} \;=\; (\beta_i - \beta_j) \ln \mu \,, \tag{4.11}$$

where β_i, β_j are the volumes of the symmetry groups G_i, G_j in the logarithmic representation. In a context of group of renormalization, we would call them the beta-function coefficients of the symmetry groups. Since all masses and couplings unify at the Planck scale, in expression 4.11 we have considered the additive bare value α_0 to be the same for all of them. This holds if we identify μ with $\mathcal{T}$, the age of the universe. Pulled back to the exponential picture the ratios of masses become then:

$$\frac{m_i}{m_j} \;=\; \alpha(G_{ij}) \;=\; \mathcal{T}^{\beta_i - \beta_j} \,. \tag{4.12}$$

In order to obtain the masses, we must therefore obtain the "beta-functions" β_i, β_j. According to our discussion, we cannot compute them using the rules of ordinary field theory: here we are interested in the full, non-perturbative beta functions. We can proceed as follows: we can determine the ratios of these beta-functions if we know the ratios of the phase-space volumes. For instance, if a projection reduces by one half the phase space, the beta-function will be one half of the initial one. On the other hand, the phase space volumes can be determined if we know the spectrum of interactions of the various particles (i.e., the pattern of figure 1), but the important point is that, in this framework, this in principle is equivalent to knowing the chain of projections, $\equiv$ symmetry reductions, giving origin to the sector a certain massive state belongs to.

Once obtained the ratios of beta-functions, in order to get all their values we must fix one of them. To this purpose, we must consider that the mass of the state with maximal, unbroken symmetry does not change with time, it is a constant. Maximal symmetry, and therefore also supersymmetry, implies in fact that masses either vanish (as also the cosmological constant does), or they do not renormalize out of the initial value at the Planck scale: among the preserved symmetries, there is in fact also time reversal, so that masses do not run. In this case, we have:

$$m_{\text{max. symm.}} \;=\; \frac{1}{2}\mathcal{T}^{\beta_{\max} - \frac{1}{2}} \;=\; \text{Constant} \,, \tag{4.13}$$

where we have considered to build the mass of the states over the minimal momentum in this space, the square-root scale, and we have set the normalization of the mass as a function of the inverse of a proper time to be 1/2, as according to the Heisenberg's Uncertainty Principle. The condition 4.13 implies $\beta_{\max} = \frac{1}{2}$. The normalization coefficient 1/2 implies that at Planck time ($\mathcal{T} = 1$) any mass is 1/2, which corresponds to the mass of a black hole, given by the relation: $2M = R$ with the identification $R = \mathcal{T} = 1$. This means that the normalization consistent with the Uncertainty Principle is consistent with the only possible mass value at Planck time: when the universe is of Planck size, there can only be a "particle" large as much as the universe itself, and the mass must be the one of a black hole extended as much as the universe. From 4.12 we see that the overall normalization is the same for all the states. Being established through a consistency argument running back to the Planck size/time, it must be the normalization of $SU(3)$-, $SU(2)_{\rm w.i.}$- and $U(1)_{\rm e.m.}$-singlets. We conclude therefore that all these masses are expressed as:

$$m = \frac{1}{2}\,\mathcal{T}^{\beta-\frac{1}{2}}\;, \qquad \beta \in \{0, \frac{1}{2}\}\,. \tag{4.14}$$

What we expect to be able to normalize in this way is not the single electron's and neutrino mass, but the $SU(2)_{\rm w.i.}$-neutral combination (e, ν_e). Furthermore, what we should be able to normalize is not the pure electron's mass but that of the electrically neutral "compound" $(e, \bar{e})$. In the first case, we can say that $m_{(e,\nu_e)} \sim m_e + m_{\nu_e} \sim m_e$. In the second case, however, similarly to what happens with $SU(3)$ and the quarks, we expect $m_{(e,\bar{e})} \sim 2m_e$. Of course, analogous considerations apply to all families, and to quarks as well, because they are all charged also under $SU(2)_{\rm w.i.}$ and $U(1)_{\rm e.m.}$. If the mass of the lighter member of a $SU(2)$ doublet can in general be neglected, being much lighter than the upper one (this is true at least for leptons), the factor 2 due to the fact that we are calculating the mass of a particle-antiparticle pair must absolutely be taken into account.

4.3.1 Elementary masses

Our approach to the computation of masses starts with a first degree of approximation, consisting of a "rough" determination of the volume of the phase space of each elementary particle, as seen "at the Planck scale". This allows us to map to a logarithmic picture, where all couplings are perturbative, because they are of order one in the original picture. Further corrections of the weak coupling scale are to be expected at the present age of the universe. This leads us out of the domain of a logarithmic picture; the problem can in principle be treated as an ordinary perturbative correction, whose complete evaluation must take into account the details of every decay channel. In any case, these corrections should be of second order, with a relative magnitude proportional to the inverse ratio of the phase volume of the particle under consideration and the one of its decay products. Namely, we expect $m = m_0 + \delta m$, where $\frac{\delta m}{m} \sim \mathcal{O}\left[m_{\rm final}/m_{\rm initial}(\equiv m)\right]$ We will consider these corrections in sections 5.3–5.6.

As discussed in section 2.2.1, the phase space is divided by the particle's families into three parts, with volumes staying in ratios given by 3:2:1. Namely, in passing from one family to the other one, the phase space volume $\mathcal{V}$ undergoes a contraction $\mathcal{V} \to \mathcal{V}^{\frac{2}{3}} \to \mathcal{V}^{\frac{1}{3}}$. In order to derive the single mass steps resulting from the superposition of configurations, we start from the lightest particle, which must necessarily belong to the lightest family, namely the one with the smallest volume in the phase space. A short inspection of the combinatorial of the various shift actions tells us that, owing to the superposition of configurations, its mass does not simply correspond to the square root of the length of space. This is the ground momentum in the space of fermions, but it would be the mass of the lightest particle only if this particle were charged under an unbroken $SU(2)$ symmetry: in this construction this would in fact mean that no further shift superposes to the fundamental one neither lifting the left, nor the right moving part of the particle. Indeed, the mass of this particle turns out to be just one "step" above the ground configuration. We may consider it an "elementary $SU(2)$ step". The mass of the lightest particle is separated from the ground momentum by a factor corresponding to the volume (the strength, or coupling) of the $SU(2)$ symmetry, $\alpha_{SU(2)}$. The lightest particle of the second family is another such a step above the first one, and the lightest particle of the third family is a further $SU(2)$ step above. The reason is that the volumes of these families are by construction differentiated by the presence in their configuration of one more Z_2 shift with respect to each other, while keeping fixed all other orbifold actions. The distance between up and down of an $SU(2)_{\text{w.i.}}$ lepton pair, such as for instance electron and corresponding neutrino, is on the other hand larger than the distance between these particles and their "replicas" of the other families, because $SU(2)_{\text{w.i.}}$ pairs are related by a symmetry broken by a shift along the space coordinates, whereas the distance between analogous particles belonging to different families is given by the superposition with different weight of configurations with the same structure of space shift, simply obtained through a different action of the same amount of orbifold projections. The steps of the mass hierarchy increase therefore first through families, and then inside each family. Since the lighest particle of each family must be identified with the less interacting one, these considerations lead to the following neutrino mass relations:

$$m_{\nu_e} = \alpha^{-1}_{SU(2)} \times \frac{1}{2} \mathcal{T}^{-1/2} \,, \tag{4.15}$$

where $\mathcal{T}^{-1/2}/2$ is the square-root energy scale corresponding to the basic shift, the scale which, in the language of 4.13, corresponds to $\beta = 0$. The masses of ν_μ and ν_τ are the obtained through the ratios:

$$\frac{m_{\nu_\tau}}{m_{\nu_\mu}} \sim \frac{m_{\nu_\mu}}{m_{\nu_e}} \sim x \,, \qquad \frac{m_{\nu_\tau}}{m_{\nu_e}} \sim x^2 \,, \qquad x = \alpha^{-1}_{SU(2)} \,. \tag{4.16}$$

The next step is a further elementary $SU2)$ factor switch from the heaviest neutrino to the lightest charged lepton, the electron. Similar steps separate then the charged leptons of the three families, leading to a family-to-family sequence analogous to those between neutrinos:

$$\frac{m_e}{m_{\nu_\tau}} \sim \alpha^{-1}_{SU(2)} \,; \tag{4.17}$$

$$\frac{m_\mu}{m_{\nu_\tau}} \sim \alpha^{-2}_{SU(2)} \,; \tag{4.18}$$

$$\frac{m_\tau}{m_{\nu_\tau}} \sim \alpha^{-3}_{SU(2)} \,. \tag{4.19}$$

The mass relations of quarks in principle obey the same rule. However, quarks occur in strongly paired triplets. The relation of combinatorial factors determining the weight of their occurrence in the staple of string configurations is therefore not simply 1 : 2 : 3 as in the case of single states, but, in the logarithmic representation, $3 \times (1 : 2 : 3)$, or, in the exponential, physical space, $3 : 3^2 : 3^3$. With respect to the leptons, their weights must on the other hand be normalized by the fact of being these triplets at the strong coupling, and therefore behaving, from the point of view of the absolute weight in the phase space, as singlets (remember the breaking of the $\mathbf{4}$ into $\mathbf{1}_1 \oplus \mathbf{1}_2$, see section 2.1.5): in the logarithmic representation we have therefore a normalization factor $1/3$, corresponding to a $\sqrt[3]{}$ in the physical space. This means that the "volume" of a quark triplet as compared to the one of the lepton family, $\mathcal{V}(q_{\rm up}, q_{\rm down}, \ell)$, in the heaviest family appears to the first power, whereas those of the other two families appear rescaled as:

$$\mathcal{V}(c, s, \mu) \sim [\mathcal{V}(t, b, \tau)]^{\frac{2}{3}} \; ; \quad \mathcal{V}(u, d, e) \sim [\mathcal{V}(t, b, \tau)]^{\frac{1}{3}} \; , \tag{4.20}$$

In order to derive the mass ratios it is therefore convenient to start with the heaviest family. For similar reasons we expect that, differently from what happens for the leptons, in the case of quarks the $\alpha_{SU(2)}$ factor between the top and bottom quark separates not the masses of the single quarks, but $SU(3)$ triplets, i.e. singlets of the confining symmetry. We expect therefore a factor $1/3$ correcting the mass ratio between the top and bottom quark. The top-to-bottom mass ratio should therefore be:

$$\frac{m_t}{m_b} \sim \frac{1}{3} \, \alpha^{-1}_{SU(2)} \, . \tag{4.21}$$

The mass separation between quarks and leptons is the consequence of the breaking of the $\mathbf{4}$ of each family into $\mathbf{3} \oplus \mathbf{1}$. This separates the phase space in two parts of unequal volumes. In first approximation, this separation corresponds to disentangling "one half" as compared to the usual $SU(2)$ steps, and therefore we expect the "up" of the $\mathbf{1}$ part to lie a $\sqrt{\alpha_{SU(2)}}$ factor below the "down" of the $\mathbf{3}$ part. We expect the separation factor between bottom quark and τ lepton to be:

$$\frac{m_b}{m_\tau} \approx \frac{1}{3} \frac{1}{\sqrt{\alpha_{SU(2)}}} \, . \tag{4.22}$$

Again a $1/3$ factor is needed in order to account for the passage from $SU(3)$ singlets to free quarks. Altogether, the top-tau mass ratio is:

$$\frac{m_t}{m_\tau} = \frac{m_t}{m_b} \times \frac{m_b}{m_\tau} \sim \frac{1}{3} \, \alpha^{-1}_{SU(2)} \times \frac{1}{3} \frac{1}{\sqrt{\alpha_{SU(2)}}} \, . \tag{4.23}$$

According to 4.20, the analogous separation for the first family should read:

$$\frac{m_u}{m_e} \sim \frac{1}{9} \left(9 \frac{m_t}{m_\tau} \right)^{\frac{1}{3}} \, , \tag{4.24}$$

where we have first removed the $\frac{1}{3} \times \frac{1}{3}$ factor from the m_t/m_τ ratio, and then reintroduced it after having taken the third root. This was required by the fact that these normalization factors, accounting for the passage from free quarks to $SU(3)$ singlets, don't enter in the contraction of phase sub-spaces. Putting all the informations together, we conclude that the phase-space sub-volume of the charged particles of the first family, $\mathcal{V}(u,d,e)$, should be given by:

$$\mathcal{V}(u,d,e) \; = \; 9 \, \frac{m_u}{m_{\nu_\tau}} \; \sim \; \alpha_{SU(2)}^{-1/3} \, \left(\sqrt{\alpha_{SU(2)}}\right)^{-1/3} \times \alpha_{SU(2)}^{-1} \, . \tag{4.25}$$

The second power of this volume gives finally $\mathcal{V}(c,s,\mu)$. To summarize, the mass ratios are:

$$\frac{m_\mu}{m_{\nu_\tau}} \; \sim \; \alpha_{SU(2)}^{-2} \, ; \tag{4.26}$$

$$\frac{m_s}{m_{\nu_\tau}} \; \sim \; \frac{1}{3} \, \left(\sqrt{\alpha_{SU(2)}}\right)^{-2/3} \, \alpha_{SU(2)}^{-2} \, ; \tag{4.27}$$

$$\frac{m_c}{m_{\nu_\tau}} \; \sim \; \frac{1}{9} \, \alpha_{SU(2)}^{-2/3} \, \left(\sqrt{\alpha_{SU(2)}}\right)^{-2/3} \, \alpha_{SU(2)}^{-2} \, ; \tag{4.28}$$

$$\frac{m_\tau}{m_{\nu_\tau}} \; \sim \; \alpha_{SU(2)}^{-3} \, ; \tag{4.29}$$

$$\frac{m_b}{m_{\nu_\tau}} \; \sim \; \frac{1}{3} \, \left(\sqrt{\alpha_{SU(2)}}\right)^{-1} \, \alpha_{SU(2)}^{-3} \, ; \tag{4.30}$$

$$\frac{m_t}{m_{\nu_\tau}} \; \sim \; \frac{1}{9} \, \alpha_{SU(2)}^{-1} \, \left(\sqrt{\alpha_{SU(2)}}\right)^{-1} \, \alpha_{SU(2)}^{-3} \, . \tag{4.31}$$

These relations are completed by:

$$m_{\nu_\tau} \; \sim \; \frac{1}{2} \, \mathcal{T}_{\text{(string)}}^{-\frac{1}{2}} \times \left[\alpha_{SU(2)}^{-1}\right]^3 \, . \tag{4.32}$$

$$m_{\nu_\mu} \; \sim \; \frac{1}{2} \, \mathcal{T}_{\text{(string)}}^{-\frac{1}{2}} \times \left[\alpha_{SU(2)}^{-1}\right]^2 \, . \tag{4.33}$$

$$m_{\nu_e} \; \sim \; \frac{1}{2} \, \mathcal{T}_{\text{(string)}}^{-\frac{1}{2}} \times \alpha_{SU(2)}^{-1} \, . \tag{4.34}$$

Owing to the particular choice of normalization of the vacuum, which is done for an electrically neutral state, we expect that these mass formulae give us twice the mass of any state, i.e. the mass of the particle-antiparticle pair. The values we obtain in this way are just the "bare" values of the mass ratios, the first step in the approximation, which must be improved by time-dependent corrections, in order to account for finer details of the phase spaces. In the next sections we will pass to the explicit evaluation of all the mass values. We will there discuss also the corrections to the bare expressions, required by an improved description of the details of the string configuration. This is particularly necessary in order to discuss the masses of the second family, strongly affected by the "stable" mass scale of the universe, the mean scale discussed in section 4.3.6, and the quarks of the first family. As we will see in section 5.2, what happens in this case is that consistency of the vacuum implies an exchange in the role of the up and down quark, so that the up quark is effectively the lighter one of the $SU(2)$ doublet.

4.3.2 The $SU(2)$ coupling

In order to compute masses, what remains to know is the beta-function of the broken $SU(2)$ group which constitutes the basic ingredient of mass ratios. In order to determine the $SU(2)$ beta-function, we will derive the volume occupied by the broken $SU(2)$ by counting the volume reductions produced by the various projections we have applied in order to reach the configuration of minimal symmetry. The counting must go back till the $\mathcal{N}_4 = 2$ point, the point at which, in our construction, the gauge beta-function vanishes. The interval of values quoted in the r.h.s. of 4.14 is therefore covered by seven Z_2 projections: the further twist breaking supersymmetry to $\mathcal{N} = 0$, the four independent rank-reducing shifts to produce the **4** out of the **16** in each family (the third family corresponds to a sector given by the product of projections, therefore it is not the result of independent operations), and the two shifts along the transverse space-time coordinates. Each one of these projections corresponds to (the breaking of) an $SU(2)$ factor. At the highest breaking, we have a double-shifted space with typical ground momentum $\mathcal{T}^{-\frac{1}{4}}$ (alternatively, we can as in Ref. [48] consider the typical ground momentum $\mathcal{T}^{-\frac{1}{2}}$ in a configuration at the $SU(2)$ extended symmetry point). The "beta-function coefficient" (or better "exponent") of $SU(2)$ is then $\frac{1}{7}$ of the full exponent, fixed by considering that for $\mathcal{N}_4 = 2$ the beta function must vanish (no renormalization at all) and it is obtained by taking the product of $SU(2)$ factors from the scale of the maximal shifting. The sum of the beta-function coefficients of these factors must therefore be equal to the span of the exponents of the scales from the scale of maximal shifting to zero, namely the interval $[0 - 1/4]$ (if we consider as starting point the lowest mass scale, we must take into account that this is an $SU(2)$ extended symmetry point). In any case, according to expression 4.14, the beta-function exponent of $SU(2)$ is:

$$\beta_{SU(2)} = \frac{1}{7} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{28} . \tag{4.35}$$

The coupling of $SU(2)$ is therefore:

$$\alpha_{SU(2)} = \mathcal{T}^{-\frac{1}{28}} . \tag{4.36}$$

Using the value of the age of the universe given in appendix A, we obtain that, at the present day, $\alpha^{-1}_{SU(2)} \sim 147$. If more precisely we use the age of the universe suggested by the agreement with neutron's mass, eq. 4.52 (i.e. $\sim 5.038816199 \times 10^{60} \mathrm{M}_{\mathrm{P}}^{-1}$, see Appendix A), we obtain:

$$\alpha^{-1}_{SU(2)} \sim 147.2 \; (147.211014) . \tag{4.37}$$

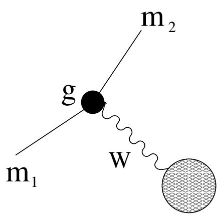
Being obtained through a counting of projections in the Z_2 orbifold approximation, 4.36 probably constitutes only an approximation of the real value of the beta function. However, we expect the relative correction to $\beta^{-1} = 28$ to be small, of the order of the relative magnitude of an inverse root of the age of the universe, as compared to this integer value:

$$\beta^{-1} \approx 28 + \mathcal{O}\left(\alpha^{-1} \mathcal{T}^{-1/p_\beta}\right) , \tag{4.38}$$

which should reflect in a similar correction also for the coupling α ($\alpha^{-1} \to \alpha^{-1}(1+\mathcal{O}(1/\mathcal{T}^{1/p_\beta}))$.

4.3.3 The $U(1)_\gamma$ coupling

In order to obtain the coupling of $U(1)_\gamma$, the electromagnetic group, we don't need to determine the absolute fraction of a group factor within the full symmetry group: we can determine the ratio of the $U(1)_\gamma$ and $SU(2)$ phase spaces, or equivalently the ratio of the two exponents, by counting the charged matter states, and subtracting the number of gauge bosons. We can justify this if we consider that the latter contribute somehow "in opposite way" to the matter-to-matter scattering amplitudes. Consider a diagram corresponding to a matter-to-matter transition:



$$g_{\rm eff} \simeq \frac{g}{M_W^2} \tag{4.39}$$

For what concerns the initial and final matter states, we have that the larger the mass ratio between initial and final state, the larger is the decay amplitude. The boson mass appears instead at the denominator in the expression of the effective coupling, and suppresses the process. A better way to see this is to consider that in the logarithmic picture the most entropic vacuum appears as effectively supersymmetric, with $\mathcal{N}_4 = 2$ extended supersymmetry [17]. As seen from the logarithmic picture, the beta-function exponent is a $\mathcal{N}_4 = 2$ beta function coefficient. In this case $b = T(R) - C(G)$. An equal number of matter states and gauge bosons, transforming in the same representation, corresponds to an effective $\mathcal{N}_4 = 4$ restoration, a situation of non-renormalization, with vanishing beta-function exponent. The phase space coefficient of $U(1)_\gamma$ is proportional to:

[17] The logarithmic picture is obtained through an artificial decompactification of the coupling of the theory. In some dual representation of this phase the theory may therefore appear supersymmetric. We already discussed that $\mathcal{N}_4 = 1$ is a fake, unstable configuration consisting of the projection onto just the perturbative part of the spectrum of a theory which, non-perturbatively, is non-supersymmetric. In the case we are interested in a correct understanding of the beta-functions, mapping to a $\mathcal{N}_4 = 2$ logarithmic representation is more appropriate than to a "fake" $\mathcal{N}_4 = 1$. This is what we have done in section 4.3.3, in order to understand the role played by matter states and gauge bosons in the evaluation of the $U(1)_\gamma$ beta-function as compared to the $SU(2)$ beta-function. In this representation, there is no "parity restoration" in the sense of the $SU(2)_{(\mathrm{R})}$ bosons coming to zero mass. The fact that $\mathcal{N}_4 = 2$ supersymmetry doesn't have a chiral matter spectrum (hypermultiplets include the conjugate states of fermions) simply means that we must expect a doubling of the matter states, due to the fact that both the left and right moving part of a matter state get paired to a conjugate. On the other hand, this is not a problem, because this picture is just a useful representation, in which we can understand certain properties, that must however be appropriately pulled back to the physical picture. In the computation of the beta-function coefficients we don't need to consider this doubling of degrees of freedom, because this is also related to an effective disappearance of one of the projections.

3(families) $\times$ 2($SU(2)$doublets) $\times$ $(\mathbf{1}+\mathbf{3})$(leptons + quarks) $\times$ 2(left + right chirality) [= 48] $-$ 1(gauge boson) = 47. Notice that, in the counting, we have considered that *all* the matter states are charged under $U(1)_\gamma$. Three states, the three neutrinos, are however uncharged. However, the electromagnetic charge is simply "shifted" from the central value $(\frac{1}{2}, -\frac{1}{2})$, but the traceless condition is preserved. As a result, the charge is only "rearranged" among the states: some states result more charged, some less. In total, the strength of the renormalization is the same as with a traceless $U(1)$ with a charge equally distributed among all the states. This is true in first approximation, when all masses are considered vanishing.

The beta-function coefficient of $SU(2)$ is proportional to 48 (the same effective number of states as for $U(1)_\gamma$) minus 3 (the number of gauge bosons), i.e. 45, where the coefficient of proportionality is the same as for $U(1)_\gamma$. The ratio of the two coefficients is therefore:

$$\frac{\beta_{U(1)}}{\beta_{SU(2)}} = \frac{47}{45}\,. \tag{4.40}$$

Using 4.36 and 4.40, and the scale $\mu = \mathcal{T} \sim 5.038816199 \times 10^{60}\mathrm{M}_\mathrm{P}^{-1}$, the present age of the universe 4.52, adjusted on the neutron mass, we get:

$$\alpha_\gamma^{-1} \sim 183.777867\,. \tag{4.41}$$

This has to be considered as a "bare" value of the coupling, not an effective coupling in the field theory sense. We will discuss in sections 4.4 and 5.4 how this value should be "run back" to obtain the effective coupling to be compared with the value experimentally measured at a certain scale.

4.3.4 The $SU(2)_{\mathrm{w.i.}}$ coupling

Determing the coupling of the $SU(2)$ of the weak interactions is even more problematic than determining α_γ. The point is that for us this symmetry is not spontaneously broken in the classical sense, and we cannot compute the beta-function coefficient in an effective theory with unbroken gauge symmetry. In the usual field theory approach, the $SU(2)$ acting on just one of the two helicities transforms only half of the matter degrees of freedom, and therefore, if we neglect the contribution of the gauge bosons, its beta-function coefficient turns out to be one half of that of a "full" gauge group, namely, with a vectorial coupling to the matter currents. This is however true as long as the matter states are massless (on the other hand, once they acquire a mass, the gauge symmetry is broken). Massive states consist of both left and right degrees of freedom. From the point of view of the volume occupied in the phase space, although interacting with just their left-handed part, massive matter degrees of freedom count as much as left + right chiral states. The volume occupied by $SU(2)_{\mathrm{w.i.}}$ is therefore intermediate between the situation of pure chiral gauge symmetry acting on massless states, therefore on half the space of the degrees of freedom, and a full vectorial interaction. Since couplings are defined as volumes obtained from averages over a superposition of configurations, we can consider that our coupling lies "in between" the two situations. Since matter states acquire a mass through a shift that reduces by half the logarithmic volume of the space (resulting therefore in a square-root scaling law), we can expect that the logarithmic volume occupied by $SU(2)_{\rm w.i.}$ is the mean value between the one of the vectorial interaction (acting therefore on the same number of degrees of freedom as the massive matter states), and the one of the pure chiral interaction, viewed as acting on massless states:

$$\beta_{SU(2)_{\rm w.i.}} \approx \frac{1}{2}\left(1 + \frac{1}{2}\right) \times \frac{1}{28}\,. \tag{4.42}$$

The present-day value of the inverse of the $SU(2)_{\rm w.i.}$ coupling should therefore be:

$$\alpha_w^{-1} \approx \mathcal{T}_0^{-\left(\beta_{SU(2)_{\rm w.i.}}\right)} \sim 42.26\,, \tag{4.43}$$

where we have used the estimate of the age of the universe 4.52. The value 4.43 is roughly a factor 4.4 smaller than the inverse electromagnetic coupling, given in 4.41. Also this number has to be considered a "bare" value, to be corrected in the way we will discuss in section 4.4.

4.3.5 The strong coupling

In our framework, the $SU(3)$ colour symmetry is always broken, and in principle there is no phase in which the strong and electromagnetic interactions can both at the same time be treated as gauge field interactions. In particular, there is no (under-Planckian) phase in which the strongly coupled sector comes down to a "weak" coupling, which merges with the other couplings of the theory to build up a unified model with a unique coupling, taking up the running up to the Planck scale. For us, the strongly coupled sector is strongly coupled at any sub-Planckian, i.e. field theory, scale. The coupling α_s will always be larger than one:

$$\alpha_s \sim \mathcal{T}^{-\beta_s}\,, \qquad \beta_s < 0\,. \tag{4.44}$$

Indeed, the representation in terms of an $SU(3)$ gauge symmetry is something that belongs more to an effective field theory realization than to the non-perturbative scenario we are considering. Namely, in our case we just know that, as soon as the space is sufficiently curved (i.e. symmetry sufficiently reduced), we have the splitting into a weakly and a strongly coupled sector, mutually non-perturbative with respect to each other.

In order to derive the exponent β_s, we must proceed as in section 4.3.2, by computing the amount of symmetry reduction, this time however in the "S-dual" representation. When seen from the point of view of the full space, this duality is indeed a T-duality . This is basically the reason why the coupling increases as the temperature of the universe decreases (or equivalently its volume increases). We expect therefore that, when seen from the point of view of a dual picture, the coupling arises in a vacuum which underwent the same amount of symmetry reduction as in the dual $SU(2)$ case of section 4.3.2. However, the space-time coordinates feel a "contraction" which is T-dual to the one experienced in the picture of the electro-weak interactions. Therefore, when referred to the time scale of the electroweak picture, the beta-function exponent, the coefficient β_s, should be 1/4 of its analogous given in 4.35. Of course, as seen from the electroweak picture, the sign is also the opposite (an inversion in the exponential picture reflects in a change of sign of the logarithm). We expect therefore:

$$\beta_s = -\frac{1}{4} \times \frac{1}{28}\,. \tag{4.45}$$

In other words, the strong coupling in itself should run as:

$$\alpha_s \sim (\mathcal{T}_{\rm dual})^{\frac{1}{28}} , \tag{4.46}$$

but the time scale $\mathcal{T}_{\rm dual}$ is related to $\mathcal{T}$ by an inversion *times* a rescaling. As the value 4.36 can be seen as the "on-shell" value at the matter scale $1/\sqrt{\mathcal{T}}$, logarithmically rescaled by a factor 1/2 with respect to the un-projected time scale $\mathcal{T}$, the scale $\mathcal{T}_{\rm dual}$ feels an inverse logarithmic rescaling, $(1/2)^{-1}$. In total, as compared to the square-root scale, it has a logarithmic rescaling by a factor 4. In order to refer the value of the strong coupling to the square-root scale of the electroweak picture, we must therefore take its fourth root. The present-day "bare" value of the strong coupling is therefore [18]:

$$\alpha_s|_{\rm today} \sim \left[(\mathcal{T}_0)^{\frac{1}{4}} \right]^{\frac{1}{28}} = \mathcal{T}_0^{-\beta_s} \sim 3.48 \, . \tag{4.47}$$

As in the case of α_γ and α_w, in order to be compared with the coupling currently inserted in scattering amplitudes also this one has to be "run back" in the way we will discuss in section 4.4 .

4.3.6 The neutron mass

We want now to discuss the physical meaning of the mean mass scale introduced in Ref. [2]:

$$\langle m \rangle = \frac{1}{2} \mathcal{T}^{-\frac{3}{10}} \, . \tag{4.48}$$

According to its definition as a sum over all the states, $\langle m \rangle = \sum_i \langle i | m | i \rangle$, the contribution to the mean value should be provided by the asymptotic stable mass eigenstate(s) of the theory. These are not necessarily elementary mass/interaction eigenstates: in general they will be compounds. Usually, one thinks at the singlets of the strong interactions, because the theory is constructed as a perturbative vacuum around the zero value of the electromagnetic and weak couplings. Here however the situation is different: a finite, non-perturbative functional mass expression, valid at any value of the space-time volume, corresponds to a regime in which not only the strong interactions are non-perturbative, but also the electroweak interactions cannot be considered weak: the perturbative description of electro-weak interactions is an approximation, whose degree of accuracy increases with the age of the universe [19]. The true free mass eigenstates are neutral to <u>both</u> the strong and electroweak interactions. The mean value 4.48 corresponds therefore to the average value of the mass of stable matter in the universe. Since the time dependence of gauge couplings is much milder than that of masses:

$$\alpha \sim \frac{1}{\mathcal{T}^{1/p}} \, , \quad m \sim \frac{1}{\mathcal{T}^{1/q}} \, , \quad p \gg q \, , \tag{4.49}$$

already outside of a close neighborhood of the Planck scale we rapidly fall into a regime in which the gravitational interaction is weak, while all other interactions are still strong.

[18] Also in this case we don't need a high precision in the estimate of the age of the universe.

[19] The behaviour of these couplings will be discussed in sections 4.3.2 and 4.3.3.

This is the regime of interest for our problem (at precisely the Planck scale the configuration becomes trivial). In this phase, the only state neutral under strong, electromagnetic and weak interactions is a compound made out of a neutron-antineutron pair at rest, and their decay products, i.e. the proton-electron-neutrino/antiproton-positron-antineutrino system. At the "strong" limit of the weak coupling, family mixings can be neglected because one can assume that all heavier particles have rapidly decayed to the ground family. As it happens for stable matter, the decay probability of the neutron is compensated by an equal probability of the inverse process of neutrino capture, and the system is stable under weak interactions. It is invariant under charge reversal, and stable under electromagnetic interactions as well. This is the *only* singlet under all the above interactions, and therefore the only mass eigenstate at finite volume. At the present age of the universe, the volume of space-time is anyway large enough to assure weakness of the electro-weak interactions. This compound is therefore not necessarily a "bound state", as it has presumably been at earlier times. We expect expression 4.48 to account for the mass of the "composite bound state", i.e. roughly twice as much as the mass of the neutron-antineutron pair. Therefore:

$$m_{\rm n} \,=\, \frac{1}{4} < m >= \frac{1}{8}\mathcal{T}^{-\frac{3}{10}} \,. \tag{4.50}$$

By inserting in 4.50 the current value for the age of the universe, as obtained by extrapolating data of experimental observations within the theoretical framework of Big Bang cosmology, we obtain a value quite close to the neutron mass. Namely, from 4.50 and a central value of the age of the universe $\sim 12.75 \times 10^9$ yrs, ($\sim 5 \times 10^{60} \mathrm{M}_{\rm P}^{-1}$, see Appendix A) we obtain:

$$m_{\rm n} \,\approx\, 937\,\mathrm{MeV}\,, \tag{4.51}$$

quite in good agreement with the value experimentally measured, 939.56563 ± 0.00028 MeV [49]. A more correct analysis would require a new derivation of the value of the age of the universe completely *within our framework*. On the other hand, within our theoretical scheme one can reverse the argument, and take the mass of the neutron as the most precise measurement of the age of the universe. In this case, we obtain as its actual value:

$$\mathcal{T}_0 \,=\, 12.62028271 \,\times\, 10^9\,\mathrm{yr}\,. \tag{4.52}$$

The fact that our mass formula gives as average mass the mass of the neutron is nicely in agreement with what we would expect from a universe behaving as a black hole. According to the common astrophysical models, a black hole is in fact the step just following the "neutron star" phase of a star at the end of its life. Our considerations of above suggest that the universe can be roughly thought as a kind of neutron star at the point of transition to a black hole.

4.3.7 The apparent acceleration of the universe

We are now in the position to come back to the issue of the apparent acceleration of the universe. We have seen that the average mass of the stable matter scales with time as:

$$m \,\sim\, \mathcal{T}^{-3/10}\,. \tag{4.53}$$

If we take this mass as the reference for the atomic mass scale, we derive that the above behaviour induces an apparent shift in the frequencies of the light emitted at different distances from the observer, i.e. at different ages of the universe, due to the different scale of the atomic energy levels:

$$\frac{\tilde{\nu}_1}{\tilde{\nu}_2} = \left(\frac{\mathcal{T}_2}{\mathcal{T}_1} \right)^{\frac{3}{10}} . \tag{4.54}$$

Once "subtracted" from the bare red-shift 3.8, this gives an apparent, effective red-shift $z_{\text{app.}}$:

$$1 + z_{\text{app.}} = \left(\frac{\nu_1}{\nu_2} \right)_{\text{observed}} = \left(\frac{\mathcal{T}_2}{\mathcal{T}_1} \right)^{\frac{7}{10}} , \tag{4.55}$$

as if the universe were expanding with rate $\tilde{R} \sim \mathcal{T}^{7/10}$, normally expected for a matter dominated era.

At the base of what is considered an experimental evidence of the accelerated expansion of the universe is the observed acceleration in the time variation of the red-shift effect. Here, this effect receives a different explanation, in terms of accelerated variation of ratios of mass scales, and therefore of observed emitted frequencies. Indeed, one may question whether a pure expansion of the metric is observable. In the classical approach, the expansion occurs at the level of the overall scale factor of the space part of the Robertson-Walker metric:

$$ds^2 = dt^2 - R^2(t) \left[d\vec{x}^2 \right] . \tag{4.56}$$

From a physical point of view, the scale factor $R(t)$ precisely defines the speed of light, obtained from the condition $ds^2 = 0$, therefore $dx/dt = 1/R$. The classical argument is that the Robertson-Walker metric is the metric of the cosmological evolution, not the metric of local physics. This saves things from a formal point of view, but is not satisfactory from a physical point of view: saying that there is an expansion of the overall scale of the metric is equivalent to saying that there is an expansion of the scale in which space lengths are measured in terms of time length. In other words, saying that there is such an expansion means that there is an expansion (more precisely a contraction) of the speed of light. Suppose we want to compare wavelengths between present time and a time at which the scale was 1/2 of the present one. From a physical point of view, what we observe is radiation produced by atomic transitions, and we compare wavelength keeping fixed the period of the light wave. Since in the past time lengths are contracted by 1/2 with respect to today, during each period of the wave light travels twice as much as today. Therefore, the same atomic transition generates a photon with twice the wavelength as today. However, if the space scale is contracted, also energies are different. Energies scale in fact as inverse of lengths (consider for instance the electric potential, $V = e^2/R$). In our specific example, this means that energies were doubled, and, according to $E = h\nu$, also frequencies were doubled, or equivalently periods were halved. The same atomic transition produced therefore photons with twice the frequency, or half the period, as compared to today. This fact, combined with the fact that the speed was doubled, implies that, for the same physical phenomenon, the effective wavelength was the same as today. Any such an overall scale of the metric is therefore physically unobservable.

4.4 *The effective couplings: part 1*

The couplings α_γ, $\alpha_{\rm w.i.}$ and α_s derived in section 4.3.2 and 4.3.3 and 4.3.5 run with time, and therefore with an energy scale: they are the couplings at a specific age of the universe. The values we obtained do not however correspond to the actual value of the physical coupling. In order to obtain the latter, we must run them up to the appropriate scale, using a finite-volume regularization. Our renormalization prescription is that we keep on imposing that the neutron's mass is the one given as in 4.50: we treat the neutron's mass as an already *renormalized* value, and consider the relation 4.50 as an "on shell prescription" which we use in order to fix the regularization.

4.4.1 *The electromagnetic and weak couplings*

To start with, in this section we consider the correction to the weak "gauge" couplings. In the representation in which elementary particles are defined, namely in the logarithmic picture, the effective gauge couplings are corrected according to:

$$\alpha_i^{-1} \approx \alpha_i^{-1}|_0 + b_i \ln \mu/\mu_0 \, , \tag{4.57}$$

where b_i are appropriate beta-function coefficients, and μ is the scale of the process of interest (this can be the electron mass in the case of the fine structure constant). Since the couplings scale as powers of the age/size of the universe, and therefore meet at 1 at the Planck scale, in first approximation we can assume that, in the effective representation of the physical configuration, couplings run logarithmically with an effective beta-function such that, starting from their "bare" value at the actual $\mathcal{T}^{-1/2}$ scale, they meet at zero at the Planck scale:

$$\alpha_i^{-1} \approx \alpha_i^{-1}|_0 + b_i^{(\text{eff.})} \ln \mu/\mu_0 \, , \tag{4.58}$$

with $b_i^{(\text{eff.})}$ such that:

$$b_i^{(\text{eff.})} \ln \mu_0 = \alpha_i^{-1}|_0 \, , \tag{4.59}$$

where:

$$\mu_0 \sim \frac{1}{2} \mathcal{T}^{-\frac{1}{2}} \, , \tag{4.60}$$

$\mathcal{T}$ being the age of the universe as fixed by the neutron's formula 4.50. The choice of the square root scale 4.60 as the starting scale is dictated by the fact that this is the fundamental scale of matter states, and their interactions. Matter consists of spinors and their compounds, and a spinor feels a square-root space, in that twice a spinor rotation corresponds to a true vectorial space rotation. As we will discuss in section 5.3, the exact normalization of the end scale for elementary states is 1/2 of 4.60.

Let's consider the electromagnetic coupling. The value of α_γ given in section 4.3.3 must be considered as a bare value at the scale μ_0. The fine structure constant, which for us is not really a constant, but just the present-day value of this coupling, will correspond to the value of α_γ run from 4.41 at the scale 4.60 to a scale μ_γ, the scale of the electron at rest. This is also the original scale at which historically the electric charge has been referred to. Although modern experiments are in general not performed at the electron's scale, through renormalization techniques their measurements are anyway always reduced to the electron's scale. From the point of view of our theoretical framework, this is the scale at which the "charged world" starts. Below this scale, there are the un-charged particles, and, from a classical point of view, the electric charge effectively ceases to exist. Once recalculated on the electron's mass scale, 4.41 gets corrected to:

$$\alpha_{\gamma}^{-1} \, : \, \alpha_{\gamma}^{-1}|_{\mu_0} \; = \; 183.78 \, \to \, \alpha_{\gamma}^{(0)\,-1}|_{m_e} \, \approx \, 132.85 \, , \tag{4.61}$$

where we used the value 5.36 for the electron's mass. The result 4.61 is definitely closer to the experimental value, nevertheless still quite not right, being out for an amount higher than the error in our approximations. The reason is that the value 4.61 has been calculated by assuming a perfect logarithmic running, without taking into account an important modification of the volume of the phase space of the charged matter particles around the electron and up quark mass scale, something we will do in section 5.2. We postpone therefore a detailed evaluation of the fine structure constant to section 5.4.

For what matters the weak coupling, the contact with experiment is made through the Fermi coupling constant G_F, basically the weak coupling divided by the W-boson mass squared. Any discussion about this must therefore be postponed after we have obtained this mass. However, the W mass too, in order to be calculated, requires a first order estimation of the weak coupling. Indeed, proceeding as in 4.61, we can see that also this coupling undergoes relative corrections of the right magnitude. We will come back to this coupling in section 5.9.

4.4.2 The strong coupling

In the case of the strong coupling, things are, for obvious reasons, more involved, being more model-dependent also the theoretical framework in which its effective experimental value is obtained. A possible "contact with the experiment" is the value α_s at the scale of some typical quark process, for instance the Z-boson mass in a $e^+e^- \to 4\mathrm{J}$ event: $\alpha_s(M_Z) = 0.119$ [49]. As it is usually given, g_s runs logarithmically with the scale. It seems therefore impossible to think that the "on shell" value 4.47 can be effectively corrected to a current value lower than 1 at around 100 GeV. However, not necessarily α_s must admit an effective representation in terms of a logarithmic running *at the same time*, i.e. *in the same picture* as the electromagnetic and weak couplings. Namely, although strongly and weakly coupled sectors are usually described in an effective action that accounts for all of them at the same time, attributing a logarithmic running to all of them in a unified picture, there are good reasons to believe that, especially for low energies, in the case of the effective α_s the logarithmic behaviour is only a first approximation. Indeed, electro-weak and strong coupling are mutually non-perturbative with respect to each other, and, although we don't know what the correct resummed running of α_s should be, and we can only make some speculation, we may expect that its logarithmic behaviour is only the first order approximation of a running that, in the representations in which the electro-weak couplings are linearized, is exponential. If we suppose that the amount of change computed in a certain scale interval should be viewed as the first step of an exponential correction, namely, if we suppose that it increases/decreases by a factor ~ 15 for each $\Delta\mu \approx 10^{12 \sim 13}\,\mathrm{M_P}$, then it is not impossible that, in passing from the scale $\mu_0 \sim 10^{-30}\,\mathrm{M_P}$ to $\sim 10^{-17}\,\mathrm{M_P}$ (~ 100 GeV) the value of the strong coupling passes from 4.47 to ~ 0.2. It appears therefore that an effective value of α_s lower than 1, as it is usually obtained, is not a signal of weakness of the interaction, but the result of working in a fictitious representation.

5 Present-time values of masses and couplings

Now that we have at hand the value of the $SU(2)$ coupling, we can proceed to an explicit evaluation of the masses of the elementary particles, as they can be computed using the mass formulae given in section 4.3.1. These can be considered the "bare" values. In section 5.3 we will discuss the corrections, and how, in some cases, it is even more appropriate to consider these values themselves as "corrections" of a "bare" mass scale, the stable mass scale 4.48.

5.1 Neutrino masses

We start with the less interacting, and therefore lightest, particles. According to the considerations of sections 4.1 and 4.3.1, the lightest mass level must correspond to the lightest electrically neutral particle, the electron's neutrino. Using the value of the present-day age of the universe derived from the neutron's mass, expression 4.52, we obtain the following value for the "square root scale":

$$\frac{1}{\mathcal{T}^{1/2}} \approx 4.454877246 \times 10^{-31} \mathrm{M_P} \,. \tag{5.1}$$

Following 4.16 and 4.14, the first neutrino mass should be a $\alpha^{-1}_{SU(2)}$ factor above 1/2 this scale. Furthermore, as discussed in section 4.3.1 after the expression 4.34, being related to the mass of an electrically neutral electron-positron pair through a chain of symmetry reduction factors, this procedure gives twice the mass of the neutrino, or, better, the $\nu\bar{\nu}$ mass. Using the value 4.37, we obtain therefore:

$$2\, m_{\nu_e} \approx 3.279 \times 10^{-29} \mathrm{M_P} \sim 4.0037 \times 10^{-10} \mathrm{GeV} = 0.40\, \mathrm{eV} \,. \tag{5.2}$$

After multiplication by a further $\alpha^{-1}_{SU(2)}$ factor, we obtain the second neutrino mass:

$$2\, m_{\nu_\mu} \approx 5.89 \times 10^{-8} \mathrm{GeV} = 58.9\, \mathrm{eV} \,. \tag{5.3}$$

Finally, multiplication by a further $\alpha^{-1}_{SU(2)}$ factor leads us to the tau neutrino:

$$2\, m_{\nu_\tau} \approx 8.677\, \mathrm{KeV} \,. \tag{5.4}$$

These values agree with the experimental indications of possible neutrino oscillation effects at the electronvolt scale.

5.2 The charged particles of the first family

An $\alpha^{-1}_{SU(2)}$ factor above the mass of the tau-neutrino there is the electron's mass:

$$m_e \sim \alpha^{-1}_{SU(2)} \times m_{\nu_\tau} \sim 0.639\, \mathrm{MeV} \,. \tag{5.5}$$

As discussed in section 4.3.1, this should be the mass of an electron-neutrino compound. However, as we have seen, neutrino masses are negligible in comparison to lepton masses, and with a good approximation the mass of such a compound coincides with the lepton's mass.

Continuing along the lines of section 4.3.1, from 4.25 we should be able to derive then the down and up quark masses, obtaining $m_d \sim 0.48\,\mathrm{MeV}$ and $m_u \sim 0.87\,\mathrm{MeV}$. However, this is not correct, and is contradicted by the experimental observations. The explanation has to do with the way the symmetry breaking is realized in our framework. At low energy, the $SU(2)_{\rm w.i.}$ symmetry appears as a broken gauge symmetry, with the breaking tuned by a parameter of the order of a negative power of the age of the universe. As we will see in section 5.8, the $SU(2)_{\rm w.i.}$ gauge boson masses scale in such a way that $\mathcal{T} \to \infty$ is a limit of approximate restoration of the $SU(2)_{\rm w.i.}$ symmetry. Moreover, remember that the weak force in itself is stronger than the electromagnetic force: $\alpha_w > \alpha_\gamma$ (it is called weak because for low transferred momenta, $p/M_W \ll 1$, effective scattering/decay amplitudes are suppressed by the boson mass: $\alpha_w^{\rm eff} \approx \alpha_w / M_W$). Therefore the "hierarchy" of matter is prioritarily determined by the $SU(2)_{\rm w.i.}$ charge, more than by the electric charge. As a consequence, the matter spectrum can be thought as made of two subspaces, the "up" and the "down" subspace, and the trace of the electric charge can be viewed as:

$$< Q_{e.m.} > = \sum_{\ell,\mathrm{q}} < \mathrm{up} | Q_{e.m.} | \mathrm{up} > \, + \sum_{\ell,\mathrm{q}} < \mathrm{down} | Q_{e.m.} | \mathrm{down} > , \tag{5.6}$$

where $\sum_{\ell,\mathrm{q}}$ indicates the sum over leptons and quarks. The condition of approximate restoration of the $SU(2)_{\rm w.i.}$ symmetry, and the dominance of the weak force with respect to the electromagnetic one, require that the two terms of the r.h.s. of 5.6 give an equal contribution to the total mean value of the electric charge. Otherwise, this would explicitly break the $SU(2)_{\rm w.i.}$ invariance. This imposes that the trace of the electric charge has to vanish separately on the "up" and "down" multiplets. In practice, both of them must vanish. For the validity of this argument it is essential that the weak force ends up by dominating the more and more over the electric one, and that the symmetry is restored at infinitely extended space-time; therefore, the full space must be essentially thought as separated in two $SU(2)_{\rm w.i.}$ eigenspaces. Compatibility of the theory at any finite time with the situation at the limit tells us that:

$$\mathrm{tr}\,(\nu, d) \; = \; 0 \, . \tag{5.7}$$

Since the ν charge vanishes, we have that:

$$\mathrm{tr}\,(d) \; = \; 0 \, . \tag{5.8}$$

This is only possible if, for one family, the roles of the up and down quarks, for what matters the electric charge, are exchanged, so that we have $\mathrm{tr}\,(d) \, = \, 3 \times \left(\frac{2}{3} - \frac{1}{3} - \frac{1}{3} \right) \, = \, 0$. Correspondingly, the trace of the "ups" is also vanishing:

$$\mathrm{tr}\,(\mathrm{e}, \mu, \tau, \mathrm{u}) \; = \; -1 \, - 1 \, - 1 \, + \, 3 \times \left(-\frac{1}{3} + \frac{2}{3} + \frac{2}{3} \right) \; = \; 0 \, . \tag{5.9}$$

Therefore, in one of the three quark families the role of up and down is interchanged: the quark with electric charge +2/3 is indeed the "down", while the one with charge -1/3 is the "up". In the ordinary field theory approach, this argument does not apply because the symmetry remains broken also at infinitely extended space-time [20]. Simple entropy considerations allow us to identify in which family the flip occurs. Let's consider the $SU(3)_{\text{colour}}$-singlet made out of the three quarks, one per each family, with higher electric charge, and the one made in a similar way out of the three quarks with the lower electric charge. Clearly, the first one is the most interacting singlet we can form by picking one quark from each family, and conversely the other one is the less interacting one we can form. The first must therefore also be the most massive out of all the possible $SU(3)$-singlets formed by one quark per each family, while the second one must be the lightest. The only possibility we have to achieve this condition is when the flip between charge +2/3 and -1/3 quarks occurs in the lightest family, i.e., for the quarks we usually call the up quark and the down quark. Therefore, approximately the value of the mass of the up quark is the one we computed for the lightest "down" quark states, and conversely the mass of the down quark is the one we assigned to the lightest "up". However, now the lightest quark has a higher electric charge. Namely, from charge $|Q| = \frac{1}{3}$ we pass to $|Q| = \frac{2}{3}$. This transformation is *not* a rotation of the group $SU(2)_{\text{w.i.}}$, but a pure electromagnetic charge shift. Therefore, here it does not matter that the former down had negative charge, so that the charge *difference* is $\Delta Q = \frac{2}{3} - \left(-\frac{1}{3}\right) = 1$: as far as time evolution is considered, for what matters the occupation in the phase space, or equivalently the mass, a charge conjugation is a symmetry of the theory. Therefore, what counts is the pure increase in the absolute value of the charge, which implies an increasing of the strength of the interaction of a particle, therefore the probability of interaction, and as a consequence also its volume of occupation in the phase space, that is, the mass. Indeed, doubling the charge means logarithmically doubling, i.e. squaring, the interaction probability, $P \propto \alpha \propto g^2$. Since in the present case we increase $|Q|$ by $\frac{1}{3}$ of the unit electric charge, we expect that, in passing from the electron to the lightest quark, besides the factor 4.24, we approximately gain an extra $(\alpha_\gamma)^{-1/3}$ factor [21]. The upper quark of the $SU(2)$ pair passes on the other hand from $|Q| = \frac{2}{3}$ to $|Q| = \frac{1}{3}$, but it does not acquire mass shifts (in the sense either of expansion or of contraction of its volume in the phase space) other than what already inherited by the expansion in the phase space of the lower partner quark. The two are in fact separated by an $SU(2)$ rotation, and the absolute value of their mass difference remains the same: the electric charge modification $|Q| : \frac{2}{3} \to \frac{1}{3}$ has to be seen as the result of an $SU(2)_{\text{w.i.}}$ rotation from the lower member of the pair, therefore a $|\Delta Q| = 1$ rotation, not as a charge shift by $|Q| = \frac{1}{3}$. If, in order to better compare with experimental data, instead of using the inverse of 4.41 we consider the current value of the fine structure constant at the MeV scale we will obtain in section 5.4, putting everything together we get:

$$m_d \approx 4.39\,\text{MeV}\,, \tag{5.10}$$

$$m_u \approx 2.50\,\text{MeV}\,, \tag{5.11}$$

so that:

$$\delta m_{u/d} = m_u - m_d \approx 1.89\,\text{MeV}\,. \tag{5.12}$$

[20] Notice that the usual charge assignment breaks the $SU(2)$ symmetry explicitly.

[21] No further 1/3 normalization factors are needed, because in this operation we are leaving unchanged the $SU(3)$ indices.

5.2.1 *The charged particles of the second and third family*

The masses of the charged particles of the second family are obtained from 4.26, 4.27 and 4.28. At present time, they are:

$$m_\mu \approx 94\,\text{MeV}\,; \tag{5.13}$$

$$m_s \approx 167\,\text{MeV}\,; \tag{5.14}$$

$$m_c \approx 1.539\,\text{GeV}\,. \tag{5.15}$$

The masses of the charged particles of the third family are obtained from 4.29, 4.30 and 4.31:

$$m_\tau \approx 13.85\,\text{GeV}\,; \tag{5.16}$$

$$m_b \approx 56\,\text{GeV}\,; \tag{5.17}$$

$$m_t \approx 2749\,\text{GeV}\,. \tag{5.18}$$

One can see that, up to the second family, the mass values, although all more or less slightly differing from those experimentally measured, are anyway of the correct order of magnitude. The values obtained for the third family, instead, seem to be hopelessly wrong. In the next sections we will discuss how these "bare" values get corrected by a refinement in our approximation.

5.3 *Corrections to masses*

Free elementary particles correspond to a conceptual classification of the real world, that makes sense only in the case of weakly coupled states. In our scenario, for the leptons this condition is better and better satisfied as the universe expands. Quarks are instead strongly coupled, and for us their coupling will become stronger and stronger as time goes by. In order to disentangle the properties of the elementary states as "free states", we have mapped to a logarithmic representation of the string vacuum. In this picture, owing to the linearization of the string space, it was easier to consider the ratios of the volumes occupied in the phase space by the various particles, and their interactions. However, as we pointed out, this mapping works only at times close to the Planck scale, where the "logarithmic world" becomes weakly coupled. At a generic finite time, the entire spectrum of particles is strongly coupled also in the logarithmic picture: not only the "colour" interactions are strong, but even the electro-weak symmetry can hardly be expanded around a vanishing value of the coupling. As we discussed in section 4.3.6, in such a world, the only true "asymptotic" state is neutral to all the interactions. We have identified this as a bound state made of neutron, proton, electron and its neutrino and their antiparticles. The corresponding mass scales as $\mathcal{T}^{-3/10}/2$. Strictly speaking, at finite time this is the only true "bare" state of our theory, and its mass can be used in order to set the scale of the universe. Correctly computing masses is therefore not only a matter of running the couplings determining mass ratios to the appropriate mass scale, through an iterative procedure starting from the bare values given in sections 5.1–5.2.1, but also of appropriately choosing the scale around which masses are perturbatively expanded. Masses below the $\mathcal{T}^{-3/10}$ scale should be treated as perturbations of this scale. This is the case of the proton and the neutron, which are made of up and down quarks of the first family, but have a mass much closer to the GeV scale than to the one of the quarks they are made of. By consistency, we should apply the same argument also to the electron and the neutrinos. However, since the strength of their interaction decreases with time and at present is already sufficiently small, we can safely speak of electrons and neutrinos as free states. For masses above the $\mathcal{T}^{-3/10}$ scale, things are reversed: it is rather $\mathcal{T}^{-3/10}$ which is a perturbation of the bare mass scale.

Our evaluation of masses proceeds therefore along a sequence of perturbative steps: at first we roughly determine, as in sections 5.1–5.2.1, the energy scale "at rest" of the a certain particle. Then we improve the computation by letting the mass to run from the fundamental $\mathcal{T}^{-1/2}$ scale to the specific scale, obtaining thereby an improvement in the perturbation process. Exactly knowing the running of masses entails a detailed knowledge of the interaction and decay processes the particle is involved in: these in fact decide what is the weight of a particle in the phase space. This investigation too can be viewed as part of a sequence of perturbative steps. At the first step, the logarithmic running of masses can be inferred from 4.9: by differentiation of this equation one obtains a renormalization group equation in which the running of mass ratios results to be the opposite of the running of couplings. In this case, the coupling concerned is $\alpha_{SU(2)}$, that, according to 4.40, runs more or less like the electromagnetic coupling α_γ, just a bit slower. In section 4.4 we assumed that, in first approximation, in the effective representation of the physical configuration, couplings run logarithmically with an effective beta-function such that, starting from their "bare" value at the actual $\mathcal{T}^{-1/2}$ scale, they meet at zero at the Planck scale. We can here assume that this holds for the $\alpha_{SU(2)}$ coupling too. Then, from 4.9 we derive that the relative variation of a mass along a certain scale variation is opposite to the one of the $SU(2)$ coupling:

$$\frac{\Delta m}{m} = -\frac{\Delta \alpha_{SU(2)}}{\alpha_{SU(2)}} \,. \tag{5.19}$$

Notice that, while the inverse couplings decrease to zero, and therefore couplings increase when going toward the Planck scale, masses instead decrease. This is correct, because what we are giving here are relative corrections to mass ratios, not masses in themselves (it must be kept in mind that this linearized representation makes only sense reasonably away from the Planck scale). In first approximation the mass corrections are of order:

$$\frac{\Delta m_i}{m_{\mathrm{i}}} \approx \frac{\ln \mu_0 - \ln \mu_i}{\ln \mu_0} \,, \tag{5.20}$$

where μ_i are the mass scales given in sections 5.1–5.2.1, $\mu_0 = \left(\frac{1}{2}\right)^2 \mathcal{T}^{-1/2}$, μ and μ_i are expressed in reduced Planck units, an appropriate Planck mass rescaling in the argument of each logarithm being implicitly understood. Indeed, since masses are obtained from the expressions of mass ratios, the higher mass of a pair is obtained as a function of an inverse coupling times the lower mass, which sets the scale of the process. Effectively, expression 5.20 is therefore shifted to:

$$\frac{\Delta m_{\mathrm{i}}}{m_{\mathrm{i}}} \approx \frac{\ln \mu_0 - \ln \mu_{\mathrm{i}} + \ln \mu_{\nu_e}}{\ln \mu_0} \,. \tag{5.21}$$

The first neutrino mass remains unvaried. For the other masses, we obtain:

$$m_{\nu_\mu} : \qquad 2.945\,\mathrm{eV} \to 2.739\,\mathrm{eV} \,; \tag{5.22}$$

$$m_{\nu_\tau}\ :\quad 4.3385\,\mathrm{KeV}\ \to\ 3.731\,\mathrm{KeV}\,; \tag{5.23}$$

$$m_e\ :\quad 0.639\,\mathrm{MeV}\ \to\ 0.505\,\mathrm{MeV}\,; \tag{5.24}$$

$$m_\mu\ :\quad 94\,\mathrm{MeV}\ \to\ 67.7\,\mathrm{MeV}\,; \tag{5.25}$$

$$m_\tau\ :\quad 13.85\,\mathrm{GeV}\ \to\ 8.99\,\mathrm{GeV}\,; \tag{5.26}$$

$$m_u\ :\quad 2.50\,\mathrm{MeV}\ \to\ 1.93\,\mathrm{MeV}\,; \tag{5.27}$$

$$m_d\ :\quad 4.39\,\mathrm{MeV}\ \to\ 3.35\,\mathrm{MeV}\,; \tag{5.28}$$

$$\delta m_{u/d}\ :\quad 1.89\,\mathrm{MeV}\ \to\ 1.42\,\mathrm{MeV}\,; \tag{5.29}$$

$$m_c\ :\quad 1.539\,\mathrm{GeV}\ \to\ 1.048\,\mathrm{GeV}\,; \tag{5.30}$$

$$m_s\ :\quad 167\,\mathrm{MeV}\ \to\ 118.9\,\mathrm{MeV}\,; \tag{5.31}$$

$$m_t\ :\quad 2749\,\mathrm{GeV}\ \to\ 1582\,\mathrm{GeV}\,; \tag{5.32}$$

$$m_b\ :\quad 56\,\mathrm{GeV}\ \to\ 35.3\,\mathrm{GeV}\,. \tag{5.33}$$

The only elementary particle mass we can here use for a precise comparison with experimental data is the one of the electron: neutrino masses are not yet known, and the other masses will undergo further corrections (see next sections).

The correction 5.24 must be considered as a first order correction: once determined at "order zero" the bare mass, 0.639 MeV, we have rescaled it according to 5.21, by recalculating the effective coupling on the zero order electron's scale. Now that we have the first order electron's mass scale, $\sim 0.505\,\mathrm{MeV}$, we can improve our approximation by recalculating the effective coupling at this new scale, and using this newly obtained relative mass correction in order to correct the scale of the 0.639 MeV. We obtain in this case:

$$m_e|_{2^{nd}}\,;\ 0.505\ \to\ 0.5069397\ldots\,\mathrm{MeV}\,. \tag{5.34}$$

This is still about 1% lower than the experimental value. Indeed, in order to get the *physical* mass of the electron, we must add to the "bare" mass 5.24 also the masses of the lighter states. The reason is the following. In the derivation of the mass ratios of section 4.3.1, namely proceeding from 4.9, there is the implicit assumption that all particles lighter than a certain one belong to a subspace of its phase space. Suppose we have just two particles, particle A with mass m_A, and particle B, with mass $m_B = \alpha \times m_A$, $\alpha < 1$. When we say that α is the ratio of the two volumes in the phase space, we also imply that particle A is heavier than particle B in that the space of B has been obtained by a process of symmetry reduction, by truncating the space of A. Particle A has more interaction/decay channels than B, because the space of A contains the space of B. Let's now consider the full phase space of a sub-universe consisting of A and B. The full volume is:

$$V(A) + V(B) \ = \ V(A) + \alpha\, V(A)\,. \tag{5.35}$$

Now, in our specific case A is the electron, and B is basically the τ-neutrino (we neglect here the other neutrinos, that give corrections of order $\mathcal{O}(\alpha^2)$). When we measure the mass of the physical electron, what we look at is the modification to the geometry of the space-time produced by the existence of the electron. For what we just said, deriving the electron's mass from 4.9 implies considering that, when generating the electron, we generate also the

τ-neutrino and the lighter particles. They also interact, and the modification to the whole phase space produced by the existence of the electron is indeed the full $V(A) + V(B) = V(A) + \alpha\, V(A)$. This implies that what we call the physical electron mass is the sum of the bare electron mass 5.24 *plus*, in first approximation, the mass of the τ-neutrino. This agrees with the observation we made in section 4.3, page 32, about the normalization of masses. Summing to 5.34 the ν_τ mass 5.23, we obtain then:

$$m'_e|_{2^{nd}} \,;\; 0.50694 \;\to\; 0.51057\ldots \text{ MeV}\,. \tag{5.36}$$

Of course, we can correct to the second order also the ν_τ mass, and further refine our evaluation. At this order the ν_τ mass gets increased, thereby increasing also the estimate of the electron's mass. A further recalculation of the coupling at the new scales leads on the other hand to a subsequent lowering of all masses. The approximation of the electron's mass proceeds through a converging series of "zigzag" steps of decreasing size, below and above the final value. One can easily see that in this way we better and better approximate the experimental value of the electron's mass (see [49]). However, we don't want here to go into a detailed fine evaluation of mass values, because $\sim 1\% \div 0,1\%$ is our best precision in many steps of our analysis of masses.

In general, accounting for the shifting of phase space 5.35 amounts in a small ($\mathcal{O}(\alpha^{-1})$) correction to mass values, but for the quarks of the first and second family the relative change is much higher ($\mathcal{O}(\sqrt[3]{\alpha^{-1}})$ and $\mathcal{O}(\sqrt{\alpha^{-1}})$ respectively). Once this is taken into account, the masses of the up and down quarks get further corrected to:

$$m_u: \quad 1.93\,\text{MeV} \;\to\; 2.435\,\text{MeV}\,; \tag{5.37}$$

$$m_d: \quad 3.35\,\text{MeV} \;\to\; 5.785\,\text{MeV}\,; \tag{5.38}$$

$$\delta m_{u/d}: \quad 1.42\,\text{MeV} \;\to\; 3.35\,\text{MeV}\,. \tag{5.39}$$

5.4 The fine structure constant: part 2

Let's now come back to a more precise determination of the fine structure constant. As discussed in section 4.4, the fine structure constant is the value of α_γ at the electron's scale, the scale that can be considered the reference for the operational definition of the electric charge. According to our analysis of section 4.3.1, in the phase space of all the elementary particles the phase space of the electrically neutral particles is a subspace of the space of the charged ones, in the sense that all charged particles are heavier than the neutral ones. This second subspace starts at the electron's scale. As we discussed in section 5.2, page 48, after the up-down flip in the quarks of the first family, the phase space gets further expanded by a $\sqrt[3]{\alpha_\gamma^{-1}}$ factor. This shift modifies the effective strength of the projections applied in order to get the mass hierarchy of section 4.3.1 in the sub-volume of the phase space corresponding to the first charged family. As a consequence, it modifies also the effective weight of the corresponding states, and the ratio of the effective $U(1)_\gamma$ and the $SU(2)$ beta-functions *around this scale*. The effect is that, as the states weight more, the effective running of the coupling is faster, or, equivalently, the one of its inverse slower. Namely, as the volumes of the matter phase space are expanded (or, logarithmically, shifted), the value of the electromagnetic coupling at the scale m_e effectively corresponds to the value of the coupling *without correction* at a run-back scale, $m_e^{\text{eff.}}$. The amount of running-back in the scale of the logarithmic effective coupling is equivalent to the amount of the forward shift in the logarithmic representation of the volumes of particles in the phase space. If volumes get multiplied by a factor, their logarithm gets shifted, and so gets shifted back the scale at which the coupling in its logarithmic representation is effectively evaluated. From an effective point of view, we can therefore derive the value of the fine structure constant by evaluating the electromagnetic coupling proceeding as in 4.58, but at a scale a factor $\sqrt[3]{\alpha_\gamma^{-1}}$ below the electron's scale, rather than precisely at the electron's scale as we did in 4.61 (see also Appendix C). In order to get a first rough estimate, we can use 4.61) to calculate that the effective scale μ_γ is lower than 0.511 MeV by a factor $\sim 5.102549027\ldots$. In this case we obtain:

$$\alpha_\gamma^{(1)\,-1}|_{m_e} \;=\; 137.0700548\,. \tag{5.40}$$

In order to improve our evaluation, we need a better approximation of the shape and size of the effective shift of the phase space of the first family. If we consider that the $\sqrt[3]{\alpha_\gamma^{-1}}$ shift on the up quark translates also to the down quark, the heavier in this case, we should conclude that the scale at which to evaluate $\sqrt[3]{\alpha_\gamma^{-1}}$ is around the down quark mass scale. Using the value 5.28 for the point of evaluation, we obtain:

$$\alpha_\gamma^{(2)\,-1}|_{m_e} \;=\; 137.0366167\,. \tag{5.41}$$

In order to further improve the estimation, one should then proceed as we did for the electron, by iterated steps of corrections of the down and electron scale, recalculating the $\alpha_{SU(2)}$ factors at the new scales to obtain improved estimations of m_d and of $\alpha_\gamma^{(0)}$ at the down mass scale, and so on, obtaining a series of converging "zigzag" steps. The first step corresponds to a slight increasing of the effective down mass, thereby lowering the factor $\sqrt[3]{\alpha_\gamma^{(0)\,-1}}$, eventually resulting in a slight, higher order decrease of the value of the inverse of the fine structure constant. The value 5.41 is around 0.0005 % above the current experimental one, $\alpha^{-1} \sim 137.035999\ldots$ [50], and these considerations induce to expect that the further steps of the approximation do improve the convergence toward the experimental value. However, one should not forget the major point of uncertainty, namely that we are here attempting to parametrize the effective modification of the size of the projections applied to the phase space, and therefore the value of the fine structure constant, due to a local dilatation of the phase space. A true fine evaluation of α_γ^{-1} requires first of all a better approximation of this effect. Last but not least, there is the question whether, and at which extent, a convergence toward the official "experimental value" of this parameter should be expected and desired. Beyond a certain degree of approximation, current evaluations heavily rely on QED techniques, and are extrapolated within a theoretical scheme that only at the first orders corresponds to the one discussed in this paper. A mismatch beyond this regime of approximate correspondence does not necessarily implies and indicates that the values here obtained are wrong: from a physical point of view what matters is that the effective computation of physical amplitudes produces correct results. Finally, we repeat and stress that in our framework the electric charge is time-dependent, and 5.41, possibly corrected at any desired order, only represents the present-day value of this parameter. The rate of the time variation at present time can be easily derived from the very definition. From 4.36 and 4.40 we obtain:

$$\frac{1}{\alpha}\frac{d\alpha}{dt} \;=\; \frac{1}{28}\times\frac{47}{45}\times\frac{1}{\mathcal{T}}\,. \tag{5.42}$$

In one year, the expected relative variation is therefore of order $\approx 3 \times 10^{-12}$. This is a rather small variation, however not so small when compared to the supposed precision with which α is obtained. Indeed, the most recent measurements give for its inverse a number with precisely 12 digits, a number whose variation could be observed by repeating the measurement at a distance of some years. Since however a fine experimental determination of α depends, through the theoretical framework within which it is derived, on time-varying parameters such as lepton masses etc..., it would not be an easy task to disentangle all these effects to get the "pure α time-variation". This kind of effects can be better detected when expanded on a cosmological scale, as we will discuss in section 7.4.1.

5.5 *Heavy mass corrections: the stable particles*

At large volume/large age of the universe, $1 \ll \mathcal{T} \to \infty$, electro-weak interactions are very weak, while strong interactions become stronger and stronger. The universe tends toward a configuration in which only the lightest particles of the decay chain are normally present, the probability of producing higher mass, unstable ones becoming lower and lower. The particles charged under the strong interaction tend to form bound states, "attracted" by the mean mass scale "$m_{3/10}$", defined in eq. 4.48. In practice, this means that the universe tends toward a world with matter made out of up and down quarks, electrons and neutrinos. The masses of these objects as *free* particles are those computed in sections 5.1, 5.2, and corrected in 5.22–5.33. Quarks however are not present as free particles, but form the bounds we call proton and neutron, which are stable in the sense that the neutron's decay into proton+electron+neutrino is balanced by the inverse process of electron and neutrino capture by the proton. As discussed, the stable mass scale $m_{3/10}$ corresponds to the rest energy of this system, namely the neutron + proton-electron-neutrino plus their antiparticles. If we look at the masses of the quarks and leptons constituting this system, we see that the neutron is made of an heavier quark set than the proton, and that the quark mass difference between neutron and proton is higher than the sum of the electron and neutrino masses. This means that the neutron decay into proton+electron+neutrino delivers some amount of energy. As far as the $m_{3/10}$ compound at equilibrium is concerned, this energy must be included in the account. This allows us to deduce that "at equilibrium" $m_{3/10}/2$ corresponds to four times the mass of the neutron. Furthermore, neutron and proton differ for their electromagnetic charge, i.e. for "weak" interaction properties as compared to the strong coupling; it is therefore reasonable to expect that their mass difference is basically due to the mass difference between the up and down quark. On the other hand, the $m_{3/10}$ scale is much higher than the down-up mass difference, and, as it corresponds to a stable scale, it can be considered weakly coupled. This implies that the quark mass difference can be treated as a small perturbation of the $m_{3/10}$ scale. We can therefore write:

$$m_{\rm p} \approx \frac{1}{4}\, m_{3/10} \,+\, \mathcal{O}(\delta m_{u,d})\,. \tag{5.43}$$

Indeed, since the proton is a stable particle, the difference in the volume occupied in the moduli space by neutron and proton is entirely due to the difference of the volumes occupied by the quarks they are formed of. Differently from what discussed at pages 5.3 and 5.3, a correction to the down quark mass does not require summing the mass of the lighter states, as it was the case for instance of the electron, whose bare mass had to be corrected by adding the ν_τ mass. The reason is that, in this case, once bound to form the heavy, strong-coupling-singlet compound, the lighter particle, the quark up, does not interact anymore: it does not have an independent phase space, as it was the case of the τ-neutrino. For what concerns the physical mass of the proton and the neutron, namely, as long as, like for the case of the electron, we look at the modification caused to the geometry of space-time by the existence of the proton and the neutron, the phase space of the up quark does not add to the phase space of the "bare" down quark. The quark mass difference entering in this game is therefore the (corrected) bare quark mass difference 5.29. *At the quark scale*, this is $\delta m_{u/d} = 1.42\,\mathrm{MeV}$. However, for what matters the mass difference between neutron and proton, the scale at which the quark masses have to be run is the proton/neutron scale. At this scale, once recalculated according to 5.21, the quark masses are:

$$m_u|_{E=m_n} = 1.7189\,\mathrm{MeV} \tag{5.44}$$
$$m_d|_{E=m_n} = 3.0183\,\mathrm{MeV} \tag{5.45}$$

that imply:

$$m_d - m_u|_{E=m_n} = 1.299\,\mathrm{MeV} \approx m_\mathrm{n} - m_\mathrm{p}\,, \tag{5.46}$$

in a quite good agreement, apart the usual $\mathcal{O}(1\,\%)$ mismatch, with the experimental value of the neutron-proton mass difference [49]. Notice that, in their logarithmic running, masses, and mass differences, decrease when increasing the scale. This is the opposite of what happens in the real, cosmological scaling. The point is that in the real scaling they all tend to 1 in Planck units. As it happens for the couplings, also the logarithmic corrections to the effective masses tend to the logarithm of 1, namely, to zero, at the Planck scale.

Expression 5.43 accounts with good approximation for the behaviour of the proton-to-neutron mass relation far away (i.e. well below) the Planck scale. As we get close to this scale, this approximation looses its validity.

5.6 Heavy mass corrections: the unstable particles

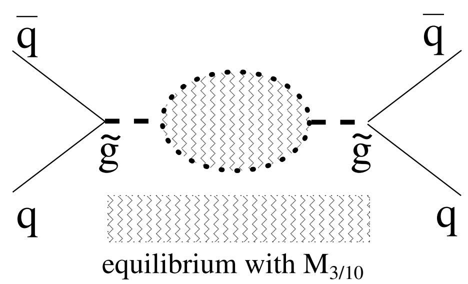
Let's now consider the particles that exist only for a short time: the leptons μ, τ and the mesons. At a sufficiently large age of the universe, these particles can be viewed as a fluctuation out of a "vacuum" characterized by the mass scale $m_{3/10}$, of which they are a perturbation. Indeed, since a perturbative approach implies working in a logarithmic picture, it is the lower mass what is going to be viewed as the "bare" value to be corrected. The masses we consider are in fact below the Planck scale, and, being everything measured in terms of this scale, higher masses are mapped into smaller ones:

$$m_1 > m_2 \quad \Rightarrow \quad |\ln m_1| < |\ln m_2|\,, \quad (|m_1|\,, |m_2| < 1\,)\,. \tag{5.47}$$

This is the $m_{3/10}$ mass itself in the case of particles with a bare mass higher than the $m_{3/10}$ scale, as is the case of the particles of the third family (the quarks top, bottom and the τ). Or it can be the mass of the particle, as is the case of the second family (charm, strange and muon). In any case, the correction is of the form:

$$M_0^2 \rightsquigarrow M^2 \approx M_0^2 \left(1 + \alpha \frac{m^2}{M_0^2} \right) , \tag{5.48}$$

where M_0 and M are the bare and the corrected mass, and m is the perturbing mass, the mass scale with which the state of mass M is in contact through an interaction with strength $\alpha = g^2/4\pi$. In the case of particles of the third family, M is the $m_{3/10}$ scale, that from the point of view of a logarithmic picture is the higher scale, and m the bare quark or lepton mass. For the second family, things work the other way around: M is the bare mass of the particle, and m corresponds to the $m_{3/10}$. When we say "the bare quark mass" we intend here something different from the usual concept of bare quark mass. As they are usually given, quark masses are in general directly derived from the mass of mesons they form, possibly subtracted of the mass of the partner quark they are bound with, and corrected within the framework of an $SU(3)$-colour symmetry based model of hadrons. Apart from the case of the up and down quarks, the quark mass turns out to be, although not really coinciding and sometimes considerably different [22], anyway of the same order of magnitude of the meson mass. In our case, bare mass means instead the value given in 5.27–5.33. The coupling α is in general the electromagnetic coupling, which provides the strongest interaction between the two mass scales. Masses enter in expression 5.48 to the second power, because this mass correction can be viewed as a propagator correction of an effective boson, as here illustrated:



where q and $\bar{q}$ stand for a quark-antiquark pair, in the simplest case for instance in a π-meson. Indeed, an expression similar to 5.48 could be considered also for the stable barions considered in section 5.5. In that case, the strongest contact between the two scales, the up and down quark scale and the $m_{3/10}$ scale, is given by the strong coupling itself, of order one. The correction ends therefore up into the $m_{3/10}$ scale itself. For the π-mesons, or the other mesons, K, C, B etc... (these last ones more or less "by definition" in direct relation to the mass of their heaviest quark), although their constituents interact strongly, this interaction involves the quarks within each meson. The strongest contact between the two scales is

[22] See for instance the case of the strange quark and the K mesons.

however given by the electromagnetic interaction, and α is basically the electromagnetic coupling.

In the case of neutrinos, their only contact with the $m_{3/10}$ scale occurs through the weak coupling. In itself, $\alpha_{\text{w.i.}}$ is even a bit stronger than the electromagnetic coupling. However, the effective strength of the interaction is of order:

$$\alpha^{\text{eff.}} \approx \alpha_{\text{w.i.}} \times \frac{m_\nu^2}{M_W^2}, \tag{5.49}$$

where $\alpha^{\text{eff.}}$ already takes into account typical energies of neutrino processes, and should not be confused with G_F, the Fermi coupling constant. The neutrino mass corrections are therefore extremely suppressed.

The correction 5.48 reduces the effective mass of the t or b quarks and the τ lepton by around one order of magnitude, producing values close to those experimentally measured. More precisely, the top mass gets corrected to:

$$m_t \to \sim 164\,\text{GeV}, \tag{5.50}$$

where, besides the value 5.32, we have also used the value of the electromagnetic coupling logarithmically corrected to the bare top scale (α_γ^{-1} : $183.78 \to 92.91$) [23]. As in the case of the electron, this value too should be corrected at higher orders, by recalculating the "bare" top mass, from the 1582 GeV of the first order, to a second order value, to be used as starting point for the correction, to be plugged in 5.48. Then, as we did for the electron, in order to catch the full phase space of the physical top particle we must add the lighter masses, the heaviest of which are the bottom, tau, and charm masses. Here too one can easily see that these higher order corrections better and better approximate the experimental value of the top mass. Let's see the first steps of this correction. First of all, we recalculate the relative mass correction, or equivalently the relative coupling correction, run at the new corrected top mass, 1582 GeV. We obtain:

$$m_t^{(0)} = 2749\,\text{GeV} \to 2749 - (2749 \times 0.42) = 1603.53\,\text{GeV}. \tag{5.51}$$

To this, we must sum the non-negligible contributions of the bottom, τ and charm bare masses, obtaining:

$$m_t' \approx 1603.53 + 35.3 + 8.99 + 1.048 = 1648.87\,\text{GeV}. \tag{5.52}$$

Of course, to be more precise we should re-correct at the second order also the bottom, τ and charm masses, something we are not doing here. Re-plugging 5.52 in 5.48, we obtain:

$$m_t'' \approx 171.07\,\text{GeV}, \tag{5.53}$$

[23] In principle, this value could be affected by the shift in the effective beta function, centered to the electron's scale, we discussed in section 5.4. However, we don't have a recipe in order to derive the full non-linear effective running of the electromagnetic coupling. We suppose that the local modification has its peak around the electron/up/down scale, and tends to vanish both toward the $\mathcal{T}^{-1/2}$ and the m_t scale. Therefore, we neglect it in this and in the following computations, already affected in themselves by possibly larger uncertainties.

quite more in agreement with the experimental value, which is around $\sim 171.4 \pm 1.7$ GeV [51]. We don't go further in the refinement of 5.53, because, to start with, we should recalculate also the bottom, τ and charm bare masses. Then, to be more precise, we should also take into account the modifications to the effective coupling and bare mass logarithmic scales, as due to the $SU(3)$ normalization factors of the quark mass ratios, 1/3 and 1/9 for the bottom and the top of each $SU(2)$ doublet. All these corrections contribute for at most $\sim 1\,\%$, therefore an uncertainty lower than the error in the experimental value of the top mass. More importantly, we must warn here that the agreement we obtain between our estimate and the experimental value has to be taken more as the indication of the plausibility of our analysis, rather than a real fine test. We are trying to evaluate the ratios of the volumes in the phase space of the particles in a rather complicated part of the spectrum, where the regions of validity of dual perturbative approaches meet. For instance, it is not completely clear whether the best approximation is obtained by summing to the top phase space the lower masses *before* the correction through the $m_{3/10}$ scale, or *after* it. Here and in the following we choose the first option. In the case of the top quark, since the top scale is well above all these scales, this does not make such a big difference. Things become however more critical when looking at the corrections to the lower masses, such as the one of the bottom quark, the τ or the charm quark.

For the bottom, the effective coupling we use is the inverse electromagnetic at the bottom scale, $\alpha_\gamma^{-1}|_b \sim 102.95$. We obtain:

$$m_b \; \to \sim 3.61\,\mathrm{GeV}\,. \tag{5.54}$$

This scale too should then be corrected in a way similar to the top mass. Adding the tau and charm masses, we obtain:

$$m_b \; \to \sim 4.57\,\mathrm{GeV}\,. \tag{5.55}$$

This value is slightly above the average experimental estimate. However, the latter is basically *extrapolated* from the B-meson width, and 5.55, although above the extrapolated value, is actually still compatible with the mass of the B-meson. A serious comparison would require a better understanding of the theoretical uncertainties underlying the entire derivation, both on the side of our evaluation of volumes in the phase space, and on the side of the experimental derivation: for consistency, the extrapolation from experimental data should be done entirely within the light of our theoretical scheme.

For the τ lepton we use a value of the electromagnetic coupling run to the lepton's scale, $\alpha_\gamma^{-1}|_\tau \sim 106.55$, and obtain:

$$m_\tau \; \to \sim 1.28\,\mathrm{GeV}\,. \tag{5.56}$$

For the further corrections to this value, analogous arguments apply also here, with the difference that, being the τ mass so close to the $m_{3/10}$ scale, the final result is more sensitive to these corrections than in the top and bottom case. For instance, at the second order the corrected bare τ mass, instead of 5.26, is $m_\tau|_{2^{nd}} \sim 9.52$ GeV, that gives 1.32 GeV. Adding the charm mass, we get a further correction by some 5%, leading to:

$$m'_\tau \; \sim \; 1.39\,\mathrm{GeV}\,. \tag{5.57}$$

As it is also the case of the quarks of this family, in particular the bottom quark, it doesn't make however sense to go on with refinements of scale evaluations, as it is already clear that something more fundamental is here missing, in order to explain the gap between the values we obtain and the so-called experimental one ($\sim 1.78\,\mathrm{GeV}$ [49]). As we said, a better understanding of the corrections to the volumes of phase spaces around the $m_{3/10}$ scale for unstable particles is in order. In the case of the bottom quark, the experimental value too is strongly affected by model-dependent considerations, and things are even more complicated.

When we pass to the second family, analogous considerations hold for the charm quark, whose mass is extremely close to $m_{3/10}$. In first approximation, by inserting the renormalized value of the electromagnetic coupling at the bare charm mass scale, $\alpha_{\gamma}^{-1}|_c \sim 113.5$, we obtain a slight decrease of the quark mass:

$$m_c \,:\, 1.048 \to 0.946\,\mathrm{GeV}\,. \tag{5.58}$$

However, as it is already evident from the τ mass evaluation of above, as the bare scale approaches the $m_{3/10}$ scale, our perturbation method starts showing its limitations. Indeed, in the case of the charm quark, it would be also possible to invert the role of bare mass and perturbing mass, using the charm bare mass 5.30 as the mass M in the expression 5.48, and, for m, the neutron mass, obtaining:

$$m'_c \,:\, 1.048 \to 1.051\,\mathrm{GeV}\,. \tag{5.59}$$

Including the strange-quark mass shift, we would obtain a light increase to:

$$m'_c \,\sim\, 1.170\,\mathrm{GeV}\,. \tag{5.60}$$

Similar considerations as for the bottom and τ masses are in order here too, and we leave any further analysis for the future.

For the strange quark and the μ-lepton, they are below the $m_{3/10}$ scale, and, as we start to get far away from it, the reliability of our estimate starts to improve again. For the strange quark, we use $\alpha_{\gamma}^{-1}|_s \sim 117.94$, to obtain, if we don't consider the μ-mass shift:

$$m_s \,\to \sim 147\,\mathrm{MeV}\,, \tag{5.61}$$

and, when including the muon mass shift:

$$m_s \,\to \sim 205.7\,\mathrm{MeV}\,. \tag{5.62}$$

A comparison with what is known as the experimental value of the strange quark mass is affected by theoretical considerations. In itself, the strange quark mass is extrapolated via $SU(3)_{\mathrm{colour}}$-related techniques from the width of the K-mesons. Surely, in the space of the K-mesons there is also the μ- channel. However, when the "bare" s-quark mass is disentangled from the total width, does this mean that also the μ- shift gets decoupled? In this case, the value to be considered for a comparison should not be the second one, 5.62, but the μ-unshifted one, 5.61. The difficulties rely here also on the fact that we are comparing *extrapolated* values, not true "experimental" ones.

Finally, for the muon we use $\alpha_\gamma^{-1}|_\mu \sim 119.42$, that leads to:

$$m_\mu \; \to \sim 109.4\,\mathrm{MeV}\,, \tag{5.63}$$

and, when including the electron mass shift:

$$m_\mu \; \to \sim 109.8\,\mathrm{MeV}\,. \tag{5.64}$$

One may notice that our mass corrections become the less and less precise as we get closer to the $m_{3/10}$ mass scale. Indeed, our approximation of the correction works better when the bare scale of the particle is far away from $m_{3/10}$, so that we can either treat the particle's scale, or the $m_{3/10}$ scale, as the perturbing or the perturbed scale. When they are close, other "non-linear" effects become important, and with our approximation we systematically obtain an overestimate for the particles with a mass below $m_{3/10}$ (muon and s-quark), and an underestimate for the particles that are above (charm, tau, (bottom ?)).

5.7 *The π and K mesons*

The π^0 mesons are bound states of the up and down quarks, that, differently from the proton and the neutron, "interact" with the $m_{3/10}$ scale through the electroweak coupling felt by their quarks, instead than directly through the strong force. As a consequence, the relation of the meson to the quark mass is given as according to 5.48, where in this case α is the electromagnetic coupling. We expect therefore:

$$\begin{aligned} m_\pi^2 \;\; &\sim \;\; \mathcal{O}(m_{\mathrm{q}}^2) \, \times \, \left\{ \alpha_{e.m.} \, \mathcal{O}(m_{3/10}^2) \; + \; \mathcal{O}(1) \right\} \\ &\approx \;\; \mathcal{O}(m_{\mathrm{q}}^2) \times \left\{ \alpha_{e.m.} \, (2m_{\mathrm{n}})^2 \; + \; \mathcal{O}(1) \right\} \, . \end{aligned} \tag{5.65}$$

This leads to a $\sim$ 100 MeV scale. As we already observed, in principle the s-quark mass, corrected by the $m_{3/10}$ scale as given in 5.62, is somehow already the effective mass "corresponding" to the K meson. It is not our scope here to enter into the details of the relation between the effective quark and meson mass, that, according to the common framework in which experimental data are interpreted (and therefore masses are derived) are supposed to be linked through $SU(3)$-colour-splitting relations. We want here only point out that, for what matters the charged mesons $\pi^\pm$ and $K^\pm$, they occupy a different phase space volume than the corresponding neutral ones; since the difference is due to the $U(1)_\gamma$ transformation properties, i.e. to the quark content, we expect the mass difference between charged and neutral mesons to be of the order of the mass difference of the component quarks. However, differently from the case of the neutron–proton mass difference, here we don't have stable particles. For proton and neutron the phase space is basically the same (in the sense that they continuously transform the one into the other), so that their differences simply reflect the differences in the properties of the bare particles they are formed of. For the π and K mesons, charged and neutral ones have instead access to completely different decay and interaction chains. Their phase spaces are therefore really different. As a consequence, although of the order of the mass difference of their quarks, the mass difference of the mesons are further modified by the modifications of the volumes of their effective phase spaces They should therefore be investigated as higher order corrections, after a recalculation of the phase spaces obtained by correcting the bare ones according to the meson interactions.

5.8 *Gauge boson masses*

According to Ref. [2], the mass of the bosons of a broken $SU(2)$ factor of a gauge symmetry is related to the masses m_1, m_2 of the particles transformed by this symmetry factor through:

$$\alpha = \frac{m_1 m_2}{M_W^2} \, . \tag{5.66}$$

Specifying this relation to the case of the $SU(2)_{\text{w.i.}}$ symmetry, we should expect that the scale of the W boson is set by the heaviest $SU(2)$ doublet, because it is the scale above which the symmetry is effectively "restored". We have thereefore:

$$\alpha \frac{3 m_t m_b}{M_W^2} \approx 1 \, , \tag{5.67}$$

where $\alpha \equiv \alpha_{SU(2)_{\text{w.i.}}}$. The factor 3 can be understood in this way: each $SU(2)_{\text{w.i.}}$ transformation rotates one quark colour; we need therefore three such rotations in order to pass from the bottom to the top quark phase space. Notice that the relation 5.67 can be viewed as the integral form of a renormalization group equation. Differentiated and mapped to a logarithmic (and therefore in general also supersymmetric) representation, it roughly corresponds to the usual expressions of the beta-function:

$$\alpha \frac{m_t \, m_b}{M_W^2} \approx 1 \quad \overset{\partial, \log}{\rightsquigarrow} \quad b \approx T(R) - C(G) \, , \tag{5.68}$$

where b is the gauge beta-function coefficient and $T(R)$, $C(G)$ are the contributions of matter and gauge, entering with opposite sign. Inserting the mass values obtained in section 5.2.1, corrected as in section 5.3, namely 5.32 and 5.33, and the value of the weak coupling 4.43, run at the bottom scale, $\alpha^{-1} \sim 24,1$ [24], we get:

$$M_{W^{\pm}} \sim 83.4 \, \text{GeV} \, . \tag{5.69}$$

In order to obtain this mass, we used for the top and bottom mass the "bare" values of page 50, not the values after the correction that brings them to their actual experimental

[24] In principle, also the weak coupling should undergo an effective beta-function modification similar to the one of the electromagnetic coupling discussed in section 5.4. However, as discussed in section 5.4 and in the footnote at page 57, this is expected to be a local modification, that tends to vanish toward the upper end scale of the matter sector, the scale that at present time is around the TeV scale. At the W-boson scale, $\alpha_{\text{w.i.}}$ should have almost regained its "regular" value. However, we cannot exclude a slight modification toward a lower effective value, which could explain why we get a boson mass slightly higher than the experimental one. If we assume a "linear" decrease of the effect, from the MeV to the TeV scale, we should find that, if at the MeV scale the weak coupling undergoes a shift proportional to the one of the effective electromagnetic coupling: $\alpha_{\text{w.i.}}|_{1\,\text{MeV}} \to \alpha_{\text{w.i.}}|_{1\,\text{MeV}} \times (132.8/137)$, at the 80 GeV scale it should have lost $\sim 2/3$ of its effect, leading to a $\sim 83.0\,\text{GeV}$ W-boson mass (see Appendix C).

value. Indeed, the relation 5.67 involves in its "bare" formulation bare particles. As it was for the quarks, also W are unstable and their mass is corrected by their interaction with the $m_{3/10}$ scale. However, for gauge bosons things go differently than for matter states, and their corrected mass cannot simply be obtained by plugging in 5.67 the corrected values of m_t and m_b. Gauge bosons behave T-dually with respect to particles; therefore, in their case, we must use an expression like 5.48 in its T-dual form:

$$\frac{1}{M_W^2} \to \frac{1}{M_W^2} \left(1 + \alpha \times \frac{1}{M_W^2} \int \frac{d^4 p}{(p + m_{3/10})^2} \right) , \tag{5.70}$$

where $m_{3/10}$ here basically stays for the neutron's mass, and the integral is intended up to the W-boson energy. Since $M_W > m_{3/10}$, $1/M_W < 1/m_{3/10}$, and, as in section 5.6, we correct the lower (inverse) scale $1/M_W$ with the higher (inverse) scale $1/m_{3/10}$. Moreover, the effective W-boson contact interaction is not suppressed by W-boson transfer propagators, and the strongest interaction they have with the $m_{3/10}$ scale occurs through the weak coupling. Therefore, here $\alpha = \alpha_{\text{w.i.}}$. Owing to the different type of effective loop correction to the boson interaction with matter, as compared to the one of matter with matter, the term that multiplies the coupling is of order 1. We have therefore:

$$\frac{1}{M_W^2} \to \frac{1}{M_W^2} \left(1 + \alpha_{\text{w.i.}} \right) , \tag{5.71}$$

or, T-dualized back:

$$M_W^2 \to \approx M_W^2 \left(1 - \alpha_{\text{w.i.}} \right) . \tag{5.72}$$

Inserting the value of $\alpha_{\text{w.i.}}$ at the W mass scale, $\alpha_{\text{w.i.}}^{-1}|_{M_W} \sim 23.46$, we obtain:

$$M_{W^\pm} \to \approx 81.6 \,\text{GeV} . \tag{5.73}$$

All the above expressions, 5.70, 5.71 and 5.72, neglect terms of order $\mathcal{O}(\alpha^2)$ (according to [50], the fit of the current experimental values of the $W^\pm$ mass is around 80.399 ± 0.023 GeV; its difference with respect to our estimate is therefore of the order of the corrections we are neglecting).

The mass of the Z boson cannot be directly derived in a similar way, by simply substituting m_t to m_b in 5.67: when $m_1 = m_2$ the symmetry is not broken, and the boson is massless! In first approximation, we expect the Z mass to be of the order of the mass of the W bosons. What distinguishes the mass of the Z boson from the one of the chiral $W^\pm$ bosons is that the Z boson acquires a "right moving" component: while the charged bosons interact only with a left-handed chiral current, the neutral boson has now a certain amount of coupling with a right-moving current. Since the Z mass is related to the volume it occupies in the phase space, the disagreement between the W and the Z mass is tuned by the strength of $SU(2)_{\text{w.i.}}$ as compared to $U(1)_Z$. In order to derive the mass of the Z boson, consider therefore once again the relation 5.67 this time with Z, W^- and W^+ replacing respectively the top, bottom quarks and the W boson: in this case we view the process as a transition between W^- and Z, produced by an element of the "group" $SU(2)_{\text{w.i.}}/U(1)_Z$ (more precisely not a group but a coset). The coupling g is now the "coupling" of $SU(2)_{\text{w.i.}}/U(1)_Z$. More precisely, since, as we discussed in section 4.3.3, the relation between "width" in the phase space and mass, in the case of gauge bosons, is the inverse with respect to the case of matter states (higher probability = lower boson mass), the relation 5.67 has to be "T-dualized" in the space of couplings; namely, "S-dualized". The effective coupling which enters in this relation is therefore the inverse of the "coupling" g^* of $SU(2)_{\text{w.i.}}/U(1)_Z$. This on the other hand is precisely what we should expect. If we set:

$$\alpha_{SU(2)_{\text{w.i.}}} = \alpha^*_{SU(2)_{\text{w.i.}}/U(1)_Z} \times \alpha_{U(1)_Z} \,, \tag{5.74}$$

being the $U(1)_Z$ coupling smaller than the one of the unbroken group, we obtain that $\alpha^* > 1$, and the relation 5.67 must be dualized in order to reduce to the ordinary weak coupling. The mass of the W boson virtually mediating the process appears on the other hand in the denominator, as in 5.67. Since we are considering a transition between bosons instead of fermions, what we obtain is a relation for the square of masses (mass terms are of the type $m^2\phi^2$ instead of $m\psi^2$):

$$\left(\frac{M_Z}{M_W} \right)^2 \approx \alpha^*_{SU(2)_{\text{w.i.}}/U(1)_Z} \,, \tag{5.75}$$

and, using the relation 5.74,

$$M_Z \sim \sqrt{\frac{\alpha_{SU(2)_{\text{w.i.}}}}{\alpha_{U(1)_Z}}} \, M_W \,. \tag{5.76}$$

In order to obtain $\alpha_{U(1)_Z}$ we can proceed as in section 4.3.3, this time by determining the fraction with respect to the volume occupied by $SU(2)_{\text{w.i.}}$ at the place of $SU(2)$. This means that the coupling of $U(1)_Z$ should stay to the coupling of $U(1)_\gamma$ in the same ratio as the coupling of $SU(2)_{\text{w.i.}}$ stays to the one of $SU(2)$. Therefore, we expect:

$$\frac{\alpha_{U(1)_Z}}{\alpha_{SU(2)_{\text{w.i.}}}} \approx \frac{\alpha_{U(1)_\gamma}}{\alpha_{SU(2)}} \,. \tag{5.77}$$

At present time, 4.37 and 4.41 and the W-boson mass 5.73 tell us that the Z boson mass should be approximately:

$$M_Z \sim 1.127 \, M_W \quad \approx 91.96 \, \text{GeV} \,. \tag{5.78}$$

If we proceed as in the footnote at page 61, by assuming a linear decrease of the local correction to the effective beta-function, this time of the electromagnetic coupling discussed in section 5.4, till its vanishing at the top scale of the charged matter phase space, 5.18, we get that at the 80 GeV scale the shift should have been reduced to around 1/4 of its size, producing a relative modification of the electromagnetic coupling at this scale of a factor ~ 1.00791, leading to a modification of the ratio 5.76 by a factor 1.00394553, i.e. a Z to $W^\pm$ mass ratio:

$$\frac{M_Z}{M_W} \; : \; 1.127 \; \to \; 1.132 \,, \tag{5.79}$$

a number that should be compared with the experimental ratio of these masses, ~ 1.134 [50]. Owing to the theoretical uncertainties implicit in our derivation, it does not make sense to refine the calculation, although it seems that the linear approximation of the effective beta-function is not quite far from the real behaviour.

Let us now consider the present-time values of the electromagnetic and the weak coupling, α_γ, $\alpha_{\rm w.i.} \equiv \alpha_W$, given in 4.41 and 4.37 ([25]), and the total width of the Z boson, given by 5.76: $\alpha_Z = \alpha_W \times (M_W/M_Z)^2$. Their *numerical* relation can approximately be written as:

$$\sqrt{\alpha_\gamma} \approx \sqrt{\alpha_{\rm w.i.}}\,\sin\theta\,; \tag{5.80}$$

$$\sqrt{\alpha_Z} \approx \sqrt{\alpha_{\rm w.i.}}\,\cos\theta\,, \tag{5.81}$$

where $\cos^2\theta \approx M_W^2/M_Z^2$. The angle θ can therefore be identified with the Weinberg angle, $\theta \sim \vartheta_w$. Indeed, since the Z boson has a larger width than the W boson only because it has a part of non-chiral interaction similar to the one of the photon, these relations say that from an effective point of view we have reproduced the first order of the electroweak gauge sector of the effective action of the Standard Model (except from the Higgs sector, of course: we don't have a Higgs field). The degrees of freedom we have obtained and their interactions can therefore be parametrized in a similar way, namely with interaction terms of the type $g\, J_\mu^{\pm} W^{\mp\,\mu}$ and $\frac{g}{\cos\vartheta_w}\left(J_\mu^0 - \sin^2\vartheta_w J_\mu^{e.m.}\right) Z^\mu$. The Z^μ term precisely says that the Z boson has total width $\alpha_Z^{\rm eff} \sim \frac{1}{4\pi}\frac{g^2}{\cos^2\vartheta_w}(1-\sin^2\vartheta_w)^2 = \alpha_w\cos^2\vartheta_w$. We stress however that in our case the relation 5.80 holds only at the numerical level, it is not a true functional relation. In our theoretical framework the gauge interactions are only an effective first order parametrization of what results from 1.1, 1.2.

5.9 The Fermi coupling constant

We are now in a position to make contact with the experimental value of the weak coupling. This is measured through the so-called Fermi coupling constant G_F, a dimensional ($[m^{-2}]$) parameter defined as the effective coupling of the weak interaction at low transferred momentum [26]:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{\pi\alpha_w}{2M_W^2}\,. \tag{5.82}$$

From section 5.8 we know that we can identify $\alpha_{\rm w.i.}$ with the usual weak coupling α_w of the literature. Inserting our results for the W-boson mass, 5.73, and the value of the weak coupling at the W-boson scale, given at page 62, we obtain:

$$G_F|_{M_W} = 1.4221 \times 10^{-5}\,\mathrm{GeV}^{-2}\,. \tag{5.83}$$

As it was for the case of the fine structure constant, once again we are faced with the problem of understanding what is the meaning of a physical quantity, whose value is always related to a certain experimental process at a certain scale. From an experimental point of

[25] In our theoretical framework, the ratio of these couplings remains the same at any scale.
[26] Low means here negligible when compared to the W-boson mass.

view the Fermi coupling is obtained by inspecting the pion into muon decay. The effective renormalization of G_F to the pion–muon scale is obtained in our framework in the same way as for the other couplings, namely treating G_F as a generic coupling, whose behaviour is represented through an effective linearization as in 4.58. The relative variation from the W to the μ or π scale [27] is of order:

$$\frac{\Delta G_F}{G_F}|_{M_W \to m_\pi} \approx 0.81 \,, \tag{5.84}$$

and we get:

$$G_F|_{\pi/\mu} \approx 1.1519 \times 10^{-5}\,\mathrm{GeV}^{-2} \,, \tag{5.85}$$

a value about 1 % away from the effective experimental value [49]. The percent is on the other hand the order of the precision we have in our estimate of the W-boson mass, and as a consequence we cannot hope to get something better for the Fermi coupling.

[27] Within our degree of approximation, it does not make such a difference the choice of one scale or the other, between muon and pion.

6 Mixing flavours

As one can expect, in our approach also the mixing of quark flavours in weak decays must be considered in the light of the volume occupied by the various decay channels in the phase space of all possible configurations. The usual classification into families, and the Lagrangian one derives for an effective action, are here justified only by their "statistical" convenience. As a matter of fact, there are no transitions in principle forbidden, but only rare as compared to other ones. The experimental observation that mass eigenstates are not weak-interaction eigenstates is traditionally encoded in a matrix $V_{\rm CKM}$, the Cabibbo-Kobayashi-Maskawa matrix, which encodes all the information about the "non-diagonal" propagation of elementary particles. It is defined as the matrix which rotates the base of "down" quarks of the $SU(2)$ doublets, allowing to express the current eigenstates in terms of mass eigenstates:

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} . \tag{6.1}$$

$V_{\rm CKM}$ accounts for the mixing among different generations, as well as for a CP violating phase. Despite the elegance of the formal treatment, and the intriguing relation between number of quarks and the existence of a phase, from the point of view of the Standard Model the entries of the CKM matrix remain external inputs, chosen to fit experimental data: there seems to be no deep reason why a mixing of quark generations should exist at all, nor why there should be a phase responsible for CP violation. The ordinary theoretical treatment simply provides a parametrization of the quark mixing, for which the number of quark families results to be precisely the minimal one allowing the existence of a phase giving rise to CP violation. In the following, we will estimate the entries of this matrix, as they can be computed for an effective action derived within our theoretical framework. We will only give the absolute values of the matrix entries, namely the parameters accounting for the amplitude of the non-diagonal decay channels. In our framework, the violation of CP is not the consequence of the existence of a non-reabsorbable phase in a complex CKM matrix, but originates from the general breaking of any kind of symmetry and parity due to the superposition 1.1, as a consequence of the implied non-invariance of the time evolution under time-reversal, both at the cosmological and local physics levels. For a discussion, we refer the reader to [4].

According to our previous discussion, the ratios between entries of the CKM matrix should be of the same order of the mass ratios, normalized to the full decay amplitude. Mass ratios correspond in fact to "couplings": $m_f / m_i \sim \alpha_{i \to f}$, accounting for ratios of subspaces of the phase space. If α_{ab} is the coupling for the flip from family a to family b, the decay amplitude of a $a \to b$ flavour changing decay is expected to be proportional to α^2_{ab}. In order to make contact with the ordinary description of the mixing mechanism, we must consider that, as it is defined, the CKM matrix is unitary, and collects the information about flavour changing, subtracted of any dependence on masses: in expressions of amplitudes, this dependence is carried by other terms. This allows to normalize the matrix in such a way that, owing to the fact that off-diagonal elements are much smaller than diagonal ones, the diagonal elements are close to 1:

$$|V_{ud}|,\ |V_{cs}|,\ |V_{tb}| \approx 1\,, \tag{6.2}$$

and, with a good approximation,

$$|V_{us}| \approx |V_{cd}| \tag{6.3}$$
$$|V_{ub}| \approx |V_{td}| \tag{6.4}$$
$$|V_{cb}| \approx |V_{ts}| \tag{6.5}$$

As for the computation of masses, a detailed evaluation of the CKM matrix entries would require taking into account all processes contributing to the determination of the phase space. Here we want just to make a test of reliability of our scheme; we are therefore only interested in a first approximation. To this purpose, it is reasonable to work within the framework of the simplifications 6.2–6.5. Owing to these simplifications, we can restrict our discussion to the off-diagonal elements $|V_{ts}|$, $|V_{td}|$ and $|V_{cd}|$. A direct, non-diagonal $t \to s$ decay should have an amplitude of order m_s/m_t, normalized then through m_b/m_t in order to reduce to the scheme 6.2. A rough prediction for V_{ts} is therefore:

$$V_{ts} \approx \frac{m_s}{m_b} \sim \frac{0.147\,\mathrm{GeV}}{3.6\,\mathrm{GeV}} \sim 0.04\,, \tag{6.6}$$

where we have used the values 5.61 and 5.54. Similarly, we obtain:

$$V_{td} \approx \frac{m_d}{m_b} \sim 0.001\,, \tag{6.7}$$

and

$$V_{cd} \approx \frac{m_d}{m_s} \sim 0.027\,. \tag{6.8}$$

While 6.6 basically agrees with the commonly expected value of this entry (see Ref. [49]), 6.7 and 6.8 are away by a factor ~ 4 in the first case, and ~ 8 in the second. An adjustment of the value is not a matter of "second order" corrections. Here the problem is that for these mixings, experimental results are mostly obtained through branching ratios of meson (π, K) decays. In these quark compounds, the strong non-perturbative resummation is highly sensitive to the GeV scale. Indeed, an experimental value $|V_{cd}| \sim 0.22$ seems to be much influenced not by the mass ratio of the bare quarks, but of the K and π mesons:

$$V_{cd} \approx \frac{m_\pi}{m_K} \sim \mathcal{O}(0.22)\,. \tag{6.9}$$

Although in a lighter way, the meson scale seems to modify also the ratio of the bottom to down quark transition. As we already said, here it is not a matter of determining a physical quantity: only decay amplitudes are physical, the CKM matrix doesn't have a physical meaning in itself. It is therefore crucial to see how do we refer to this effective tool: how much "resummation" we want to attribute to a correction to be applied to "bare" decay amplitudes computed from a "bare" CKM matrix, and how much of it we prefer to already include in the CKM matrix. As long as the final products we consider are just meson amplitudes, the two approaches are equivalent.

By comparing eqs. 6.8 and 6.9, we are faced with something at the same time reasonable and which nevertheless sounds somehow odd. On one hand, the fact that 6.9 gives a higher ratio is not surprising: it is in fact quite natural to think that a heavier particle has a larger decay probability than a lighter one. On the other hand, when applied to the $|V_{cd}|$ transition, this argument seems to lead to a contradiction: the basic degrees of freedom of a K- and π-mesons are the quarks; nevertheless, the Kaon has a larger decay probability than the quarks it is made of. Indeed, it is not in this way that the enhancement of the V_{cd} entry due to the passage from quarks to mesons has to be interpreted. The free quark "does not exist", Pions and Kaons are the lightest strong-interaction singlets containing the d and s quark. Once inserted in the computation of a decay amplitude, the values we are proposing for the entries of the CKM matrix must be corrected by some overall "form factor", of the order of m_K/m_s for the initial state, and of m_π/m_d for the final state. In practice, this is equivalent to the introduction of an "effective" CKM matrix entry, $V_{cd}^{\text{eff}} \sim (m_K/m_s)/(m_\pi/m_d) \times V_{cd}$. This rescaling eats the factor ~ 8 of disagreement between our prediction and the usual value of this entry, as reported in the literature.

Differently from the case of $|V_{cd}|$, $|V_{ts}|$ turns out to be in agreement with what reported in the literature, because the latter is derived by unitarity from $|V_{cb}|$, measured through $B \to D$ decays. Both these mesons have a mass of the same order as the b and c quark respectively. To be more precise, in these cases the quark mass itself, as is given in the literature, corresponds to the "corrected mass", basically coinciding with the mass of the meson of which it constitutes the heaviest component. The matrix entry is therefore "by definition" almost the same as the "bare" one.

The case of the $|V_{td}|$ (and V_{ub}) entries is even more involved, being much higher the uncertainties in the experimental derivation of the transition elements. In our framework, neutrinos are massive, and we expect that the CKM matrix has a leptonic counterpart. The "leptonic CKM" entries should however be more suppressed, as a consequence of the fact that all the three neutrinos are lighter than the lightest charged lepton, and their spaces have a higher separation. According to the leptonic mass values derived in section 5.1, we expect at present time approximately:

$$
\begin{aligned}
V_{\text{CKM}}^{\text{leptons}} &\equiv \begin{pmatrix} V_{e\nu_e} & V_{e\nu_\mu} & V_{e\nu_\tau} \\ V_{\mu\nu_e} & V_{\mu\nu_\mu} & V_{\mu\nu_\tau} \\ V_{\tau\nu_e} & V_{\tau\nu_\mu} & V_{\tau\nu_\tau} \end{pmatrix} \\
&\approx \begin{pmatrix} \sim 1 & \sim 0.007 & \sim 0.00005 \\ \sim 0.007 & \sim 1 & \sim 0.007 \\ \sim 0.00005 & \sim 0.007 & \sim 1 \end{pmatrix} . \qquad (6.10)
\end{aligned}
$$

Non-diagonal lepton decays are therefore more difficult to observe than those of quarks, perhaps more difficult to detect than neutrino masses themselves.

7 Interacting theory

What we have done till now is to consider the string theoretical representation of 1.1 in order to derive the spectrum and the properties in the limit in which the elementary particles can be isolated as asymptotic states. As discussed in [2], this representation can be constructed in the ordinary terms of the various dual string constructions only once gravity is basically "decoupled", which means the geometry of the base of the fibered space is flattened. Of course, in the string construction gravity is present through the graviton, but this is dealt with as one of the various fields of the spectrum, in first order decoupled from the other degrees of freedom. Flattening the space means factorizing the base, so that string theory appears to have the same physical content at each point of the three-dimensional space. The superposition 1.1, and its non-perturbative string counterpart 1.2, consist however of a sum of configurations in which different things happen in different places. Said differently, in the superposition the energy distribution is not homogeneous (the symmetry of space is broken). Unfortunately, we have at present no tools to investigate the pure non-perturbative regime of string theory, besides the simple comparison of dual constructions, which anyway correspond to limit cases, with either vanishing or extremely strong coupling. The traditional approach to string theory has developed a set of tools enabling to compute various scattering amplitudes. Although important in themselves, these tools cannot be used in our theoretical framework in order to compute terms of the interacting theory. In our context string theory does not compute densities, and therefore terms of an effective action, but global quantities. Moreover, the states one wants to identify as the asymptotic representation of free particles or fields are superpositions of states of a staple of configurations, and it is therefore not clear what kind of vertex operator they should correspond to. We can nevertheless derive some phenomenological aspects in an approximate way. We will consider here some significant cases in which the power of our approach can be seen explicitly.

7.1 The 125 GeV resonance at LHC

Let us start by considering the scattering of a particle/antiparticle pair (it can be an $\ell\bar{\ell}$ scattering, such as those of the old LEP, or a proton-antiproton pair, such as those occurring at LHC). We want to understand within our framework how it occurs that, when the rest-frame center-of-mass energy of the pair attains a value corresponding to the mass of a new particle (or field), the scattering amplitude gets enhanced. Namely, we want to understand why do we have a resonance. In the phase space, the weight of a configuration with particle 1 at position $\vec{x}_1$ and particle 2 at position $\vec{x}_2$ is the product of the weights of the two single configurations (like probabilities, weights are always < 1 because they are normalized to the total partition function, and, since they are related to volumes of symmmetry groups and group cosets, like probabilities they compose multiplicatively). Consider the moment in which $\vec{x}_1 \simeq \vec{x}_2$, and $E_{\rm tot} = E_1 + E_2 \lessapprox m_3$, where m_3 is the mass of a new particle of the spectrum, and compare it with the moment $\vec{x}_1 = \vec{x}_2$, $E_{\rm tot} = m_3$. While in the first case we have a product of two systems, each one with its complete, independent symmetry group, at the resonance point, from the point of view of 1.1, which only knows about energy distributions, there is no distinction with respect to the configuration of a *single* particle at rest with mass m_3. There is therefore a strong increase of the weight of the phenomenon in the phase space of the configurations: now there are more possibilities to produce the very same configuration.

The increase of volume in the phase space may occur also as the consequence of the total disappearance of a symmetry, such as the $U(1)$ electromagnetic symmetry. As we discussed in section 4, mass ratios correspond to coupling strengths of symmetry groups: a particle is heavier than a lighter one by a factor given by the ratio of the two corresponding symmetry groups. In the scattering of a particle-antiparticle pair of charged particles, at an energy α_γ^{-1} times higher than the sum of the energies of the two particles the configuration is at all the effects undistinguishable from the one of an energy packet with mass α_γ^{-1} times higher, with a symmetry group with volume α_γ times smaller. That means, a particle which does not possess the $U(1)$ symmetry. This does not mean this particle must exist as an asymptotic state: for the existence of a resonance it is sufficient that we created a configuration with a lower internal symmetry group, virtually equivalent to a physical particle. Notice that, no matter how close we are to the critical point, above or below this energy we have two particles, with their full internal groups, and therefore a weight in the phase space which is at most the square of the weight at the resonance (remember that weights are always smaller than 1, so that the square of a weight is smaller than the weight itself). We may call this a "bound state" of the $U(1)$ interaction. As discussed in Ref. [2], such a bound state can be expected also by considering the coulombian potential energy in a microscopic world in which the minimal distance is the Planck length. For particles of unit charge (such as electrons or protons), in Planck units the potential energy is simply $-\alpha$ (the minus sign because we have a particle-antiparticle pair), so that, in the linearized representation of the perturbative approach, the energy equation reads:

$$M \;=\; m - \alpha\,, \tag{7.1}$$

where m is the center-of-mass energy of the colliding pair, and M the resonance energy. Once pulled back from the tangent space to the physical space through exponentiation this leads to the here usual relation of mass ratios:

$$\frac{m}{M} \;=\; \alpha\,. \tag{7.2}$$

In our approach we interpret as resonance of this kind the two resonances recently detected at LHC around 125 GeV. Indeed, they are so close to almost appear as a unique resonance, that one would like to interpret as a Higgs boson signal. In our theoretical framework there is no Higgs boson as an asymptotic state: we see these two peaks as resonances of $p\bar{p} \to (pe^-)\bar{p}e^+$ and $p\bar{p} \to (p\mu)\bar{p}\bar{\mu}$, where we indicate within brackets the virtual bound state. Once inserted the effective value of the electric coupling run at the quark scale ($\alpha \sim 1/133$), we obtain a resonance at the "bound" energies $\alpha^{-1}m(pe) = 124.7\,\mathrm{GeV}$ and $\alpha^{-1}m(p\mu) = 128\,\mathrm{GeV}$. The interactions of these excited states are the same of the non-excited ones. This fact could explain why, according to the current experimental data ([45], [52], [53]), in these resonances the favoured decay channels seem to be the typical leptonic ones ($\to 2\gamma$), rather than the ones one would expect from a true Higgs boson.

The 125 GeV resonance can still be interpreted in terms of a virtual state made up of asymptotically existing states. This approach allows however to understand also energy thresholds which can hardly be interpreted in terms of usual particle and fields. An example is the cosmic background radiation, which has the typical spectrum of a black body radiation, with a temperature of about 2.8 ^0K [54, 55]. In the usual cosmological interpretation, this radiation is interpreted as being the remnant of very early processes in the universe: it would consist of photons cooled down during the expansion of the universe. At the origin they should have possessed an energy corresponding to a microwave length, as expected from energy exchange due to Compton scattering through the plasma at the origin of the universe. The low temperature would then be the effect of the cooling down of the universe due to its expansion.

In our theoretical framework it is not necessary to advocate the primordial history of the universe in order to account for the existence of a low-temperature radiation. Being a background radiation, it must not evidently come from clearly identified sources such as electronic transitions in the elements composing stars etc. Indeed, the fact that the superposition of configurations 1.2 leads to a spectrum that we can interpret in terms of the usual elementary particles and fields does not mean that the physics of the universe is completely accounted in terms of these degrees of freedom and their interactions. Like the masses of the elementary particles, also the photon energies are the result of an averaging procedure over all the configurations. As such, they do not necessarily correspond to energy levels of ordinary elementary particles: what we call photon is associated to the $U(1)$ symmetry, which is present in a full bunch of string configurations, not only those of highest entropy. The lowest energy level of an interaction of the photon with matter occurs at the level of chiral fermion fields. If one wants, it is possible to see this in terms of asymptotic states of a field-theoretical approximation as an indirect, higher loop interaction of the photon with the lightest neutrino, via an intermediate neutral-current interaction. However, it is not necessary, and perhaps not appropriate, to force an interpretation in terms of asymptotic particles. Chiral fermions are "half" of massive fermions, and as such they have a weight in the phase space which on the tangent space (the perturbative construction) is one-half of massive fermions. According to the derivation of couplings in section 4, the beta-function exponent corresponding to the amplitude of their interaction is therefore one-half, i.e. the coupling the square-root, of the one of the full fermions. In a fermionic space-time, i.e. in a "square-root" space in which the lowest momentum of a universe extended up to $\mathcal{T}$ is not $1/\mathcal{T}$ but $1/\sqrt{\mathcal{T}}$, the lowest average energy of a photon pair interacting with the vacuum is therefore:

$$\langle E_{2\gamma} \rangle \; \sim \; \sqrt{\alpha_\gamma} \, \frac{1}{\sqrt{\mathcal{T}}} \, . \tag{7.3}$$

The scale at which α_γ is fixed is determined by our way of detecting the radiation under question, namely through photon-electron interactions in our detectors. Indeed, out of the configuration of highest entropy, in which the spectrum of matter and interactions is precisely the known one, the electromagnetic $U(1)$ is not disentangled from other types of neutral current interaction. From a phenomenological point of view, it is the interaction with the electron what identifies the electric coupling, and distinguishes it from other types of neutral-current interaction, therefore singling out also what the photon is. The value of the electric coupling to be inserted in this expression is therefore the one at the electron scale. Inserting the value of α_γ at the electron's scale, derived through an effective running of the type 4.58 from the initial value 4.41 at the $\mathcal{T}^{-1/2}$ scale to the 0.5 MeV scale, $\alpha_\gamma^{-1}|_{m_e} \sim 132.3$, the value 4.52 for the present age of the universe, and converting energy into temperature through the Boltzmann constant, we obtain:

$$T_\gamma \equiv k^{-1} < p_\gamma > \; = k^{-1} E^0_\gamma \sim 2.72\,^0K\,. \tag{7.4}$$

The Gaussian tail of the resonance, leading to a black-body distribution of frequencies, is in this context the consequence of the superposition 1.1, for which the entropy sum, once restricted to the phenomenon under consideration, and thermodynamically interpreted as in section 5.1 of Ref. [2], namely through $S \sim E/T$, becomes a typical Gaussian distribution.

7.3 The fate of dark matter and the Chandra observations

A discrepancy between our framework and the common expectations is the absence in our scenario of dark matter. According to our analysis, the universe consists only of the already known and detected particles. Of course, there can be regions of the space in which a high concentration of neutrinos, which for us are massive, increases the curvature without being electromagnetically detected. But this is not going to change dramatically the scenario: there is no hidden matter acting as an extra source able to increase the gravitational force by around a factor ten over what is produced by visible matter, as it seems to be required in order to explain a gravitational attraction among galaxies much higher than expected on the base of the estimated mass of the visible stars. The problem arises in several contexts: Big-Bang nucleosynthesis, rotational speed of galaxies, gravitational lensing. All these points would require a detailed investigation, beyond the scope of this work. We will also not attempt to rediscuss a huge literature, and limit ourselves here to mention some hypotheses. The first remark is that the discrepancies between theoretical expectations and the observed effects, which are found in so different issues as primordial universe, nucleosynthesis and galaxy phenomenology, don't need necessarily to be explained all in the same way.

Let's consider the problems related to the motion of external stars in spiral galaxies, where for the first time the issue of dark matter has been addressed, and the "anomalous" gravitational lensing, with reference to the recently observed effect in the 1E0657-558 cluster [56]. It is since 1933 (Fritz Zwicky) that, by looking at the amount of red-shift in the light emitted by the stars in the wings of a spiral galaxy, it has been noticed how, differently from what expected, the rotation speed does not decrease with the inverse of the square root of the radius: it is a constant [57, 58]. Presence of invisible matter has been advocated, in order to fill the gap between the mass of the observed matter and the amount necessary to increase the gravitational force. Indeed, the expectation that the rotation speed of stars in the external legs should decrease is based on the assumption that almost the entire mass of the galaxy is concentrated in the bulge at the center of the spiral. Any star on the wings would therefore feel the typical gravitational field due to a fixed, central mass.

In the framework of our scenario, masses have been in the past higher than what they are now. Moreover, owing to the fact that, as we discuss in [1], the universe "closes up", in such a way that the horizon we observe corresponds to a "point", the space separation between objects located at a certain cosmic distance from us appears to be larger than what actually is. All this could mean that the mass of the center of a galaxy, as compared to the wings, has been systematically overestimated. It would be interesting to see, by carrying out a detailed re-examination of the astronomical observations, whether the behaviour of the center of a galaxy still requires to advocate the presence of a heavy black hole, in order to explain a gravitational force higher than what expected on the base of the estimated mass of the visible stars. In any case, it is possible that, once the downscaling of length and upscaling of masses has been appropriately taken into account, a better approximation of a spiral galaxy is the one sketched in figure 2. In part A of the picture the galaxy is (very roughly) represented with wide wings, with stars relatively "broadened" on the plane of the galaxy. Part B shows the same figure, simply with much narrower arms. In picture A the broad lines have been shadowed in a way to make evident that the higher star density of the bulge is largely due to the "superposition" of the various arms. Nevertheless, as it is clear from picture B, the problem remains basically "one-dimensional": the wings are one-dimensional lines coming out of the center of the galaxy. Under the hypothesis that all the stars have the same mass, the linear density of a wing is constant, and, once integrated from the center up to a certain radius R, the total mass M_R of the portion of galaxy enclosed within a distance R from the center is roughly proportional to R:

$$\rho = \frac{dM}{dr} \sim \text{const.} \quad \Rightarrow \quad M_R \sim \text{const} \times R\,. \tag{7.5}$$

In the expression of the gravitational potential, the linear R dependence of the mass cancels against the R appearing in the denominator (the potential remains the one of a Coulomb force). The gravitational potential energy is therefore a constant times the mass of the star in the wing. Conservation of energy implies therefore that also the velocity of the star does not depend on the radius R. We stress that this is only an approximation: it would be exact if the arms were not those of a spiral but straight legs coming out radially from the center, and under the assumption that all the stars of the bulge correspond to the superposition of the arms.

In the case of the 1E0657-558 cluster, the Chandra observatory has detected a gravitational lensing higher than what expected on the base of the amount of luminous matter. Moreover, the highest effect corresponds to two dark regions close to the cluster, rather than to places where the visible matter is more dense. In the framework of our scenario, a possible explanation could be that what is observed is the effect of a "solitonic" gravitational wave, produced as a consequence of the separation of one sub-cluster from the other one. This could increase the gravitational force by an amount equivalent to the displaced cluster mass, for a length/time comparable to the cluster size, therefore a time much higher than the few hours during which the effect has been measured ($\sim$ 140 hours). It remains that the lensing is around 8-9 times higher than what expected on the base of the amount of visible mass. However, the cluster under consideration is at about 4 billion light years away from us. This is around 1/3 of the age of the universe. This time distance is large enough to make relevant

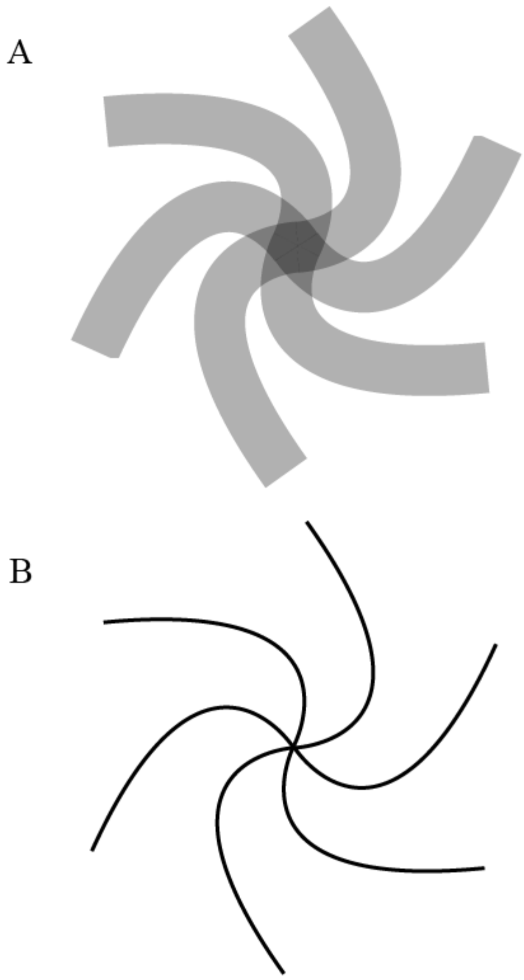


Figure 2: Picture A is the rough sketch of a spiral galaxy, in which the arms are broad and shadowed in a way to highlight the increasing mass density due to their superposition at the center. Figure B represents the same object, with the arms narrowed down, in order to highlight the one-dimensional nature of the physical problem, for what concerns the mass density.

the effects due to a change of the curvature of space-time along the evolution of the universe, as well as the change of masses. Furthermore, as we discussed above, the apparent space separation between objects located at a certain cosmic distance from us must be appropriately downscaled, in order to account for the curving up of space-time into a sphere, with the horizon "identified" with the origin. Putting all this together, we obtain that the effective gravitational force experienced on the 1E0657-558 cluster is (or, better, it was) indeed 8-9 times higher than what it appears to us on the base of the expected mass of the objects in the cluster, i.e. precisely the amount otherwise referred to dark matter.

7.4 *Cosmological constraints*

Cosmology addresses two kinds of problems for what concerns the "running back" of a theory, or an "early time" model. Namely, 1) the possible non-constancy of what are commonly called "constants", and 2) the agreement with the expected origin/evolution of the early universe (baryogenesis, nucleosynthesis etc...). In our framework, these issues are put in a light quite different from the usual perspective: there are in fact indeed no constants; therefore, a variation of couplings, masses, cosmological parameters, and, as a consequence, energy spectra, is naturally implemented. However, there is a peculiarity: all these parameters scale as appropriate powers of the age of the universe. As a consequence, a "number" close to one at present day has a very mild time dependence:

$$\mathcal{O}(1) \approx \mathcal{T}^{\epsilon} \;\Rightarrow\; |\epsilon| \ll 1\,, \tag{7.6}$$

and therefore varies quite a little with time. Oklo and nucleosynthesis bounds, being given as ratios of masses and couplings that cancel each other to an almost "adimensional" quantity, are precisely of this kind. In our case they don't provide therefore any dangerous constraint.

For what concerns the non-constancy of "constants", there are not enough data enabling to test our prediction about a time variation of the cosmological constant, whose measurement is still too imprecise. A more stringent test of the variation of parameters comes from the observations on the light emitted by ancient Quasars. In this case, the spectrum shows an "anomalous" red-shifted spectrum. This shift should not be confused with the usual red-shift, of which we have discussed in section 3.1. The effect we consider here persists once the "universal" red-shift effect has been subtracted. As an explanation, it is often advocated a possible time variation of the fine structure constant α.

7.4.1 *The "time dependence of α"*

The question of the possible time variation of the fine structure "constant" arises in the framework of string theory derived effective models for cosmology and elementary particles. Various investigations have considered the possibility of producing some evidence of this variation, or at least a bound on its size. To this regard, astrophysics is certainly a favoured field of research, in that it naturally provides us with data about earlier ages of the universe. A possible signal for such a time variation could be an observed deviation in the absorption spectra of ancient Quasars [59, 60, 61, 62]. This effect consists is a deviation in the energies corresponding to some electron transitions, which remains after subtraction of the background effect of the red-shift, and is obtained with interpolations and fitting of data.

What is observed is a decrease of the relativistic effects in the energies of the electrons cloud, with respect to what expected on the base of present-day parameters (in particular, the fine structure constant). Indeed, while the atomic spectra are universally proportional to the atomic unit $me^2 \propto m\alpha^2$, the relativistic corrections depend on the coupling α. After subtraction of the "universal" red-shift effects, their variation should then be directly related to a variation of α. In our framework, the explanation comes from considering both the scaling of α and the one of masses at the same time: going backwards in time α increases, as also the proton and the electron mass do, but *the ratio of α to the mass scales decreases*. This is the "variation of α after subtracting the universal red-shift" which is usually considered in the discussions of the literature. Namely, if we measure the variation of α with respect to the electron's mass scale (whether the true electron mass or the "reduced" mass doesn't make a relevant difference [28]), i.e. if we rescale quantities in the frame in which masses are considered fixed, we indeed observe a decrease of the coupling α. Indeed, what is done in the literature (see Refs. [60]) is not only to consider masses fixed, but to exclude from the evaluation also the effect of the red-shift. With current experimental methods, based on the interpolation of spectral data in order to find out the "background" and the variations out of it, this subtraction is somewhat unavoidable.

In order to obtain what the prediction in our scenario is, and how it compares with the literature, let's first see how the decrease of the relativistic effects, when going backwards in time, turns out to be a prediction of our framework. Consider the effective scaling of α in terms of $m\alpha^2$ units, the "universal" scaling of emission/absorption atomic energies. We have that:

$$\bar{\alpha} \stackrel{\text{def}}{\equiv} \frac{\alpha}{m\alpha^2} \approx \mathcal{T}^{\frac{1}{3}+\frac{1}{28}} . \tag{7.7}$$

The "effective" coupling $\bar{\alpha}$ scales as a positive power of the age of the universe: going backwards in time, it decreases. According to the literature, atomic energies have an approximate scaling of the type [29]:

$$E_n \approx K_n (m\,\alpha^2) + \Gamma_n\,\alpha^2 (m\,\alpha^2) , \tag{7.8}$$

where K_n and Γ_n are constants and the second term, of order α^2 with respect to the first one, accounts for the relativistic corrections. Investigations on the possible variation of α use interpolation methods in order to disentangle the second term from the first one. Since the universal part is reabsorbed into the red-shift, the relative variation should give information on just the variation of α. Expression 7.8 is of the form:

$$E_n \approx E_n^0 (1 + a_1\,\alpha^2) . \tag{7.9}$$

It is derived by considering the first terms of a field theory expansion around the fine structure constant (the electric coupling). Indeed, since we are interested in the correction subtracted

[28] In the hydrogen atom this is given by $m_e = \frac{m_e m_{\rm p}}{m_e + m_{\rm p}}$. The possibility of referring to a change of this quantity the effect measured in Ref. [60] can be found in Ref. [63, 64, 65].

[29] See for instance Ref. [60]

of the universal part reabsorbable in the red shift, we can separate the $\mathcal{O}(\alpha^2)$ term in 7.8 as:

$$K_n \;=\; (K_n \,-\, \Gamma_n) \,+\, \Gamma_n \,. \tag{7.10}$$

This allows to reduce the part of interest for us to:

$$E_n^{\rm eff} \;\approx\; E_n^0 \, (1 \,+\, \alpha^2) \,. \tag{7.11}$$

As we already observed several times along this work, perturbative expressions involving elementary particles are naturally defined and carried out in a logarithmic representation of the physical vacuum. In particular, when writing expressions like 7.8 it is intended that the coupling α scales logarithmically. An expression like 7.11 should be better viewed as accounting for the first terms of a series that sums up to an expression scaling as a certain power of the age of the universe:

$$\alpha_{\rm eff} \;\equiv\; 1 \,+\, \alpha^2 \;\approx\; 1 \,+\, \alpha^2 \,+\, \mathcal{O}(\alpha^4) \;\;\rightsquigarrow\;\; \sim \mathcal{T}^{\beta} \,, \tag{7.12}$$

where α is then not the full coupling, intended in the non-perturbative sense of 4.12, but its logarithm. According to 7.7, in the hypothesis of keeping masses fixed, this term should then effectively scale as a positive power of the age of the universe: $\beta > 0$. The exponent β can be fixed by comparing values at present time:

$$\alpha|_{\rm today} \;\approx\; \sqrt{5 \,\times\, 10^{-5}} \,. \tag{7.13}$$

We obtain therefore:

$$1 \,+\, \alpha^2 \,\approx\, \mathcal{O}(1 \,+\, 5 \,\times\, 10^{-5}) \;\approx\, \mathcal{T}^{\beta} \;\;\Rightarrow\;\; \beta \,\sim\, \mathcal{O}(10^{-6}) \,, \tag{7.14}$$

and a relative time variation:

$$\frac{\dot{\alpha}_{\rm eff}}{\alpha_{\rm eff}} \;\approx\; \beta \, \mathcal{T}^{-1} \;\approx\; \mathcal{O}\left(10^{-16} \, {\rm yr}^{-1}\right) \,. \tag{7.15}$$

This is the relative variation of the relativistic correction subtracted of the universal part (reabsorbed in the red-shift), to be compared with the results of [59], as reported also in [60]:

$$\frac{\langle \dot{\alpha} \rangle}{\alpha} \;=\; -2.2 \,\pm\, 5.1 \,\times\, 10^{-16} \; {\rm yr}^{-1} \,. \tag{7.16}$$

Since the deviation of the resummed function 7.11 from a pure exponential is of order $\alpha^4 \,\sim\, 2 \,\times\, 10^{-9}$, four orders of magnitude smaller than the dominant term, the inaccuracy in our computation is much lower than the order of magnitude of the result.

7.4.2 The Oklo bound

Data from the natural fission reactor, active in Oklo around two billions years ago, are today considered one of the most important sources of constraints on the time variation of the fundamental constants. By comparing the cross section for the neutron capture by

Samarium at present time with the one estimated at the time of the reactor's activity, one derives a bound on the possible variation of the fine structure constant, and on the ratio $G_F m_{\rm p}^2$, in the corresponding time interval. The interpretation of the experimental measurements and their translation into a bound on the variation of the capture energy resonance is not so straightforward, and depends on several hypotheses. In any case, all these steps are sufficiently under control. More uncertain is the translation of this bound on the energy variation into a bound on the variation of the fine structure constant and other parameters: this passage requires strong assumptions about what is going to contribute to the atomic energies. This analysis was carried out in Ref. [66], basically on the hypothesis that the main contribution to the resonance energy comes from the Coulomb potential of the electric interaction among the various protons of which the nucleus of Samarium consists. According to [66], after a certain amount of reasonable approximations, the energy bound translates into a bound on the variation of the electromagnetic coupling. A simple look at expression 4.36 shows that, in our scenario, the variation of this coupling over the time interval under consideration violates the Oklo bound. This bound seems therefore to rule out our theoretical framework. However, things are not so simple: the derivation of a bound on a coupling out of a bound on energies works much differently in our framework, and we cannot simply use for our purpose the results of [66]. Indeed, in our framework what varies with time is not only the fine structure constant, but also the nuclear force, and the proton and neutron mass as well. Of relevance for us is therefore not a bound on a coupling, derived under the hypothesis of keeping everything else fixed, but the bound on the energy itself [66]:

$$-0.12 \text{ eV} \; < \; \Delta E \; < \; 0.09 \text{ eV} \,. \tag{7.17}$$

In order to give an estimate of the amount of the energy variation over time, as expected in our framework, we don't need to know the details of the evaluation of the resonance energy starting from the fundamental parameters of the theory. To this purpose, it is enough to consider that, whatever the expression of this energy is, it must be built out of i) masses, ii) couplings (electro-weak and strong) and iii) the true fundamental constants (the speed of light c, the Planck constant $\hbar$, and the Planck mass $\mathrm{M_p}$). Working in units in which the latter are set to 1 (reduced Planck units), all parameters of points i) and ii) scale as a certain power of the age of the universe. As a consequence, the resonance energy itself mainly scales as a power of the age of the universe:

$$E \; \sim \; a \mathcal{T}^{-b} \,. \tag{7.18}$$

(More generically, it could be a polynomial: $E \; \sim \; a_1 \mathcal{T}^{-b_1} + a_2 \mathcal{T}^{-b_2} + \ldots + a_n \mathcal{T}^{-b_n}$. In this case, to the purpose of checking the agreement with a bound, it is enough to look at the dominant term). We can fix the exponent b by comparing the expression, evaluated using the present-day age of the universe, with the value of the resonance, that we take from [66]:

$$E \sim a \mathcal{T}^{-b} \; = \; 0.0973 \, \text{eV} \; \times \; 1.2 \, \times \, 10^{-28} \; = \; 1.2 \, \times \, 10^{-29} \, \mathrm{M_P} \,. \tag{7.19}$$

In order to solve the equation, we would need to know the coefficient a, something we don't. However, as long as we are just interested in a rough estimate, it is reasonable to assume that, since this coefficient mostly accounts for possible symmetry factors, it may affect the value of the result for about no more than one order of magnitude. Inserting the value $\mathcal{T} \sim 5 \times 10^{60} \mathrm{M}_{\mathrm{P}}^{-1}$ for the age of the universe, we obtain:

$$b \sim \frac{1}{2}\,, \tag{7.20}$$

and finally:

$$|\Delta E| \sim \frac{1}{10} E \sim 0.01 \,\mathrm{eV}\,. \tag{7.21}$$

over a time of two billion years. This is compatible with the Oklo bound, eq. 7.17.

From the Oklo data one tries also to derive a bound on the adimensional quantity

$$\beta \equiv G_F m_{\mathrm{p}}^2 (c/\bar{h}^3)\,. \tag{7.22}$$

In this case, our discussion is easier, because we know the scaling of all the quantities involved [30]. Once again, we have to deal with a quantity that scales as a power of the age of the universe. At present time, we have:

$$\beta \sim \mathcal{T}^{-b_\beta} = 1.03 \times 10^{-5}\,. \tag{7.23}$$

Inserting the actual value of the age of the universe, we obtain $b_\beta \sim \frac{1}{12}$. Over a time interval of around 1/5 of the age of the universe, this gives a relative variation:

$$\frac{\Delta\beta}{\beta} \sim 0.017\,, \tag{7.24}$$

to be compared with the one quoted in Ref. [66]:

$$\frac{|\beta^{\mathrm{Oklo}} - \beta^{\mathrm{now}}|}{\beta} < 0.02\,. \tag{7.25}$$

Both results 7.21 and 7.24, although still within the allowed range of values, seem to be quite close to the threshold, beyond which the model is ruled out. One would therefore think that a slight refinement on the measurement and derivation of these bounds could in a near future decide whether it is still acceptable or definitely ruled out. Things are not like that. Indeed, as we already stressed in several similar cases, the *entire* derivation of bounds and constraints, involving at any level various assumptions about the history of the universe and therefore of its fundamental parameters, should be rediscussed within the new theoretical framework: it doesn't make much sense to compare pieces of an argument, extracted from an analysis carried out in a different theoretical framework, with different phenomenological implications. To be explicit, in the case of the derivation of the Oklo bounds, one should reconsider all the derivation of absorption thresholds and resonances. We should therefore better take into account from the beginning the time variation of all

[30] We recall that $G_F/\sqrt{2} = g^2/8M_W^2$. Therefore, $\beta = \pi\alpha m_{\mathrm{p}}^2/\sqrt{2}M_W^2$. For times much higher than 1 in reduced Planck units, the proton mass can be assumed to scale approximately like the mean mass scale 4.48.

masses, and in particular the neutron and proton masses, as well as couplings. Perhaps a more meaningful quantity is then not anymore the pure resonance shift, but this quantity rescaled by the neutron mass. In this case, the effective variation of interest for our test is not 7.21, but:

$$\frac{\Delta(E/m_n)}{(E/m_n)} \approx \frac{\Delta\mathcal{T}^{-\frac{1}{9}}}{\mathcal{T}^{-\frac{1}{9}}} \sim 0.02\,, \tag{7.26}$$

a variation one order of magnitude smaller than 7.21 ($\Delta E/E \sim 0,1$). Analogous considerations apply also to the case of the second bound 7.24, basically equivalent to the nucleosynthesis bound.

7.4.3 The nucleosynthesis bound

Bounds derived from nucleosynthesis models are even more questionable: they certainly make sense within a certain cosmological model, but, precisely because of that, they cannot be simply translated into a framework implying a rather different cosmological scenario. Once again, the only anchor points on which we can rely are the few "pure" experimental observations, to be interpreted in a consistent way in the light of a different theory. The point of nucleosynthesis is that there is a very narrow "window" of favourable conditions under which, out of the initial hot plasma, our universe, with the known matter content, has been formed. Of interest for us is the very stringent condition about the temperature (and age of the universe) at which the amount of neutrons in baryonic matter have been fixed. As soon as, owing to a cooling down of the temperature, the weak interactions are no more at equilibrium, the probability for a proton to transform into a neutron is suppressed with respect to the probability of a neutron to decay into a proton. Owing to their short life time, comparable with the age of the universe at which the equilibrium is broken, basically almost all neutrons rapidly decay into protons, except for those that bound into ^{4}He. Nucleosynthesis predicts a fraction of ^{4}Helium and Hydrogen baryon numbers ($\sim 1/4$) in the primordial universe, which is in good agreement with experimental observations. The formula for the equilibrium of neutron/proton transitions is given by:

$$\frac{\mathrm{n}}{\mathrm{p}} = \mathrm{e}^{-\frac{\Delta m}{kT}} \sim 1\,, \tag{7.27}$$

where $\Delta m = m_{\mathrm{n}} - m_{\mathrm{p}}$. In the standard scenario, this mass difference is a constant, and the temperature runs as the inverse of the age of the universe. The equilibrium is broken at a temperature of around 0.8 MeV, when (n/p) $\simeq 1/7$. In our framework too the temperature runs as the inverse of the age of the universe, but the mass difference Δm is not a constant: all masses run with time. At large times ($\mathcal{T} \gg 1$ in Planck units), we are in a regime in which we can use the arguments of section 5.3, in order to conclude that, being the u and d quark masses much lighter than the neutron mass scale, we can consider Δm as a perturbation of $m \simeq m_{\mathrm{n}}$. In this regime, the neutron-proton mass difference is basically of the order of the constituent quark mass difference, and we have reasons to expect that it also runs accordingly. It would therefore seem that, in our case, going backwards in time, the ratio (n/p) remains lower than in the standard case, and the equilibrium 7.27 is attained at a temperature much higher. However, to the purpose of determining the processes of the nucleosynthesis, essential is not just the scaling of the equilibrium law of the neutron-to-proton ratio, but also that of the mean life of the neutron. It is the combined effect of these two quantities what determines the primordial baryon composition. In the usual approach, the neutron mean life is assumed to be constant. Being related to the neutron decay amplitude, i.e. to the volume occupied by the neutron in the phase space, in our framework this quantity too is not constant. In order to see what in practice changes in our scenario with respect to the standard one, instead of attempting to guess what the scaling behaviour of the neutron mean life could be, we can proceed by considering, instead of the pure running of the equilibrium equation, the *reduced running at fixed neutron mean life*. Certainly the mean life is constant if the neutron mass is constant. The quantity of interest for us is therefore the scaling of the mass difference, as measured in units of the neutron mass itself. According to our considerations of above, we have:

$$\Delta m_{\rm red}(\mathcal{T}) \, \equiv \, \frac{\Delta m}{m_{\rm n}} \, \sim \, \frac{\mathcal{T}^{p_{(u-d)}}}{\mathcal{T}^{p_{\rm n}}} \, , \tag{7.28}$$

where $p_{(u-d)}$ and $p_{\rm n}$ are exponents corresponding to the up-down quark mass difference and to the neutron mass respectively. This running is expected to hold not only at present time but also at a temperature of ~ 1 MeV, which is anyway much lower than the Planck scale. We can therefore compare our prediction with the standard one by simply considering the relative deviation of equation 7.27 from its standard value, as obtained by replacing the constant mass difference Δm with $\Delta m_{\rm red}(\mathcal{T})$:

$$\frac{\rm n}{\rm p} \, = \, {\rm e}^{-\frac{\Delta m}{kT}} \quad \to \quad \left(\frac{\rm n}{\rm p} \right)_{\rm red} \, \equiv \, {\rm e}^{-\frac{\bar{m}_{\rm n} \Delta m_{\rm red}(\mathcal{T})}{kT}} \, , \tag{7.29}$$

where $\bar{m}_{\rm n}$ is the *fixed*, time-independent present-day value of the neutron mass. Therefore, in the standard case $({\rm n/p})_{\rm red}$ coincides with $({\rm n/p})$. According to the mass values given in section 4, we have:

$$\Delta m_{\rm red}(\mathcal{T}) \, \approx \, \mathcal{T}^{-\frac{1}{24}} \, . \tag{7.30}$$

Considering that the time variation between the point $\mathcal{T}_f$ of the breaking of equilibrium and the present day is of the order of the age of the universe itself, $\Delta T \equiv \mathcal{T} - \mathcal{T}_f \sim \mathcal{T}$, we obtain approximately that the integral variation of $x \, \equiv \, \Delta m_{\rm red}(\mathcal{T})$ over this time interval is:

$$\Delta x \, \sim \, \frac{1}{24} x \, . \tag{7.31}$$

The "reduced value" of (n/p), $({\rm n/p})_{\rm red}$, is now modified to:

$$\left(\frac{\rm n}{\rm p} \right)_{\rm red.} : \, \frac{1}{7} \, \to \quad \sim \, \frac{1}{7} \left(1 \, - \, \frac{\ln 7}{24} \right) \, \approx \, 0.131 \, . \tag{7.32}$$

This value leads to a ratio X_4 of helium to Hydrogen of around:

$$X_4 \, \sim \, 0.232 \, , \tag{7.33}$$

still in excellent agreement with what expect from today's most precise determinations (for a list of results and references, see Ref. [49]).

Appendix

A Conversion units for the age of the universe

We give here some conversion factors from time units to Planck mass units.

$$1 \text{ year (yr)} = 3.1536 \times 10^7 \text{ s}$$

In order to convert this value to eV units we divide by $\hbar = 6.582122 \times 10^{-22}$ MeV s. We obtain:

$$1 \text{ yr} = 4.791160054 \times 10^{28} \text{ MeV}^{-1}$$

Considering that the Planck mass $\mathrm{M_P} = 1.2 \times 10^{19}$ GeV, we have also the relation:

$$1 \text{ yr} = 3.992633379 \times 10^{50} \, \mathrm{M_P^{-1}} \, .$$

The age of the universe $\mathcal{T}$, estimated to be around 11.5 to 14 billion years, reads therefore:

$$\mathcal{T} \approx \left\{ \begin{array}{l} 4.59152839 \\ 5.58968673 \end{array} \right. \times 10^{60} \, \mathrm{M_P^{-1}}$$

If instead we take the neutron mass as the most precise way of deriving the age of the universe, from expression 4.50 and the present-day measured neutron mass, we obtain:

$$\mathcal{T} \approx 5.038816199 \times 10^{60} \, \mathrm{M_P^{-1}} \quad (= 12.6202827 \times 10^9 \text{ yr})$$

B The type II dual of the $\mathcal{N}_4 = 1$ orbifold

We discuss here the type II dual construction of the $\mathcal{N}_4 = 1$ orbifold vacuum of section 2.1.1. On the heterotic side, this appears as a supersymmetric construction. We claimed that $\mathcal{N}_4 = 1$ supersymmetry exists only perturbatively, but when the full, non-perturbative construction is considered, one sees that this symmetry is broken. From the heterotic point of view, the breaking is non-perturbative, being produced by a "twist" along the coupling-coordinate around which the perturbative expansion is built. The only signal of the supersymmetry breaking is then indirectly provided by the way couplings of non-perturbative matter and gauge sectors (parametrized by perturbative fields of the heterotic string) enter in the expressions of threshold corrections of effective couplings. Namely, with the "wrong" power, as if these couplings were "inverted", from a < 1 to a > 1 value. Indeed, these couplings are parametrized by moduli only at the $\mathcal{N}_4 = 2$ level (see Ref. [17]). When the perturbative supersymmetry is reduced to $\mathcal{N}_4 = 1$, these fields are twisted. This however only means that the expectation value is not anymore a running parameter, but is fixed. We can nevertheless trace the fate of the couplings by investigating the so-called "$\mathcal{N} = 2$" sectors.

In order to follow the operation of supersymmetry breaking from the the type II side, let's first consider the starting point, the $\mathcal{N}_4 = 2$ construction. In order to make easier the investigation of the projections, it is convenient to express the degrees of freedom in terms of free fermions (Ref. [43]). In the case of type II strings, these constructions have been extensively analysed in Ref. [14]. Indeed, the cases we are referring to are embedded in a infinitely extended space-time, a situation deeply different from the one considered in this paper, where space-time is compact. However, as we have seen, in practice this reflects in a different interpretation of the results (e.g. the fact that densities become global quantities), whereas from a technical point of view the usual analysis carries over from a scenario to the other one with minor, obvious changes (the substitution of a continuum of modes along the space-time coordinates with a discrete lattice of momenta/energies). For simplicity, we use therefore here the same notation for the string modes as in the cited works. The set of all fermions is therefore:

$$F = \left\{ \begin{array}{c} \psi^L_\mu, \chi^L_I, y^L_I, \omega^L_I \\ \psi^R_\mu, \chi^R_I, y^R_I, \omega^R_I \end{array} \right\}, \quad (\mu = 1,2;\ I = 1,...,6), \tag{2.1}$$

where $\psi^{L,R}_\mu$ indicate the left and right moving fermion degrees of freedom along the transverse space-time coordinates, while $\chi^{L,R}_I$ those along the internal coordinates. $y^{L,R}_I \omega^{L,R}_I$ correspond instead to the internal fermionized bosons. The basic sets of boundary conditions are S and $\bar{S}$, which contain only eight left- or right-moving fermions, and distinguish the boundary conditions of the left- and right- moving world-sheet superpartners:

$$S = \left\{ \psi^L_\mu, \chi^L_1, \dots, \chi^L_6 \right\}, \qquad \bar{S} = \left\{ \psi^R_\mu, \chi^R_1, \dots, \chi^R_6 \right\}. \tag{2.2}$$

In order to obtain a $Z_2 \times Z_2$ symmetric orbifold, we need then the two sets b_1 and b_2:

$$b_1 = \left\{ \begin{array}{c} \psi^L_\mu, \chi^L_{1,2}, y^L_{3,\dots,6} \\ \psi^R_\mu, \chi^R_{1,2}, y^R_{3,\dots,6} \end{array} \right\}, \tag{2.3}$$

$$b_2 = \left\{ \begin{array}{c} \psi^L_\mu, \chi^L_{3,4}, y^L_{1,2}, y^L_{5,6} \\ \psi^R_\mu, \chi^R_{3,4}, y^R_{1,2}, y^R_{5,6} \end{array} \right\}. \tag{2.4}$$

These sets assign Z_2 boundary conditions and break the $\mathcal{N}_4 = 8$ supersymmetry to $\mathcal{N}_4 = 2$. The lowest entropy configuration is then obtained by further partial shifting of some states of the twisted sectors. We will not consider these further operations: they commute with the projection we want to consider in the following, namely the one that leads to the breaking of supersymmetry; considering them complicates the construction without altering the conclusions. As discussed in Ref. [14] and [17], depending on the relative phase of the projections introduced by b_1 and b_2, we obtain two mirror configurations which, according to [17], are two slices of the same model: in one we see only the vector multiplets, in the other only the hypermultiplets, of the same $U(16)$ model.

We want now to introduce another projection, dual to the $Z_2^{(2)}$. In order to understand what we have to expect from this further operation, we must take into account that 1) in order to preserve the pattern of duality with the heterotic and type I string established at the $\mathcal{N}_4 = 2$ level, i.e. the identification of the geometric moduli of the type II space with those of the heterotic space and the type I coupling moduli, also this third projection must act symmetrically on left and right movers; 2) it must twist all these moduli. On the other hand, we cannot pretend to see the extended space-time represented in a similar way in both the heterotic/type I and the type II dual: a further symmetric, independent twist on the type II space must necessarily act also on the coordinates with index "μ". This means that, in order to see the action of the heterotic $Z_2^{N=2\to 1}$ projection, on the type II side we must trade the space-time coordinates for internal ones. From the type II point of view the heterotic space-time will therefore be entirely non-perturbative, and the type II construction will look perturbatively compactified to two dimensions. Being two coordinates hidden in the light-cone gauge, we see therefore no transverse non-compact coordinates. As we discussed in section 2.1.3, representing the "11-th coordinate" of string theory in orbifolds entails its linear realization through an embedding in a two-dimensional toroidal space. The space gives therefore the fake impression of being "12-dimensional". This is however an artifact of the perturbative representation.

Compactifying the "μ" indices implies that we can now fermionize the bosons also along these coordinates. The boson degrees of freedom ∂X_μ and $\bar{\partial} X_\mu$ will be now represented as $y^L_\mu \omega^L_\mu$ and $y^R_\mu \omega^R_\mu$. (By the way, we remark that in the scenario discussed in this work, all string coordinates are always compactified. Therefore, in principle fermionization of the space-time degrees of freedom is always possible. On the other hand, when considering explicit string constructions, perturbation is always possible only around a decompactified coordinate, that works as the vanishing coupling around which to perturb. The very fact of writing a perturbative representation of a string vacuum implies the assumption that a certain limiting procedure toward a non-fermionizable point of some coordinates has been taken.)

From this two-dimensional point of view, the $\mathcal{N}_4 = 2$ type II construction contains only scalar fields: the space-time is non-perturbative, and therefore so are all indices (vector, spinor and tensor) running along space-time coordinates. The type II construction is therefore blind to the distinction between gauge and matter, whose degrees of freedom have a space-time vector or spinor index, and an internal, scalar index: only this last index is visible on the type II, and these states appear all as scalars. There is no trace of the graviton, because it bears only space-time indices. Moreover, the fields T^i and U^i, $i = 1, 2, 3$, usually appearing in one-loop expressions of threshold corrections, don't correspond now to geometric moduli of two-tori. Indeed, for any twist what remains untwisted is a four-torus. In practice, we have added a two-torus. However, as we discussed, this is an artifact of the linearization of the space; there is indeed no twelve-dimensional theory, and the appearance of the two-torus is due to an "over-dimensional" representation of a curved space with just one more coordinate, the one that served as the coupling on which to expand in the four-dimensional vacuum. The 12-th coordinate is instead a curvature. There is no surprise that, in this representation, the former moduli T^i and U^i are now multiplied by what was the coupling of the theory: its dependence was simply "frozen" by construction. For what matters duality with the heterotic construction, nothing changes, because the value of these fields was not fixed. We can recover a description in terms of moduli of two-tori by introducing independent boundary conditions for the "complex planes" (1,2), (3,4), (5,6), (7,8) (see Ref. [14] for a detailed discussion of these sets). This allows to disentangle the two-torus moduli, by factorizing the space in four two-tori. On the type II side we see then that, besides the T^i and U^i, $i = 1, 2, 3$, we have now one more field, corresponding to what was the (hidden) coupling of the four-dimensional construction. It misleadingly appears as a pair of torus moduli, T^4, U^4, respectively corresponding to the volume form and the complex structure. Owing to the symmetry of the construction under exchange of the three tori with the fourth one, a $T^4 \leftrightarrow U^4$ reflection exchanges the two $\mathcal{N} = 4$ mirror constructions (the one with only vectors with the one with only hyper multiplets). It is worth to consider more in detail this property. The "fourth torus" volume form is the product of two radii, that we call R_{11} and R_{12} for obvious reasons. The moduli T^4 and U^4 are related to these radii by: $\mathrm{Im}\, T^4 = R_{11}R_{12}$, $\mathrm{Im}\, U^4 = R_{11}/R_{12}$. As we said, one of the two radii is indeed not a real further coordinate, but a curvature. When seen from the "four dimensional point of view", an inversion of this radius corresponds to an inversion of the full string coupling. Therefore, the $T^4 \leftrightarrow U^4$ mirror exchange that relates the two constructions is an "S-duality" of the "normal representation" of the type II vacuum.

We already discussed in Ref. [17] how the heterotic construction, containing both vector and hyper multiplets, corresponds to a slice, built around a corner of the moduli space, of the "union" of both the type II mirror models. From this point of view it is therefore "self-mirror". Here we understand that this mirror symmetry is indeed a strong-weak coupling duality of the type II string, an operation which is perturbative on the heterotic dual [31]. For the rest, it is important to observe that, although we cannot explicitly verify it on the base of the carried space-time indices, all hidden, the identification of the degrees of freedom allows anyway to see the S and $\bar{S}$ as the generators of space-time supersymmetry. This time they are to be intended as a representation of the "internal part" of the supersymmetry sets.

From the above considerations, we conclude that, on the type II side, the new projection, corresponding to the step $\mathcal{N}_4 = 2 \to \mathcal{N}_4 = 1$, must be represented by a set b_3 given, up to a permutation of the three complex planes corresponding to the indices $I = 1, \ldots, 6$, by:

$$b_3 = \left\{ \begin{array}{c} \chi_{3,\ldots,6}, y^L_\mu, y^L_{1,2} \\ \chi^R_{3,\ldots,6}, y^R_\mu, y^R_{1,2} \end{array} \right\} . \tag{2.5}$$

The condition 2) of above tells us however that, differently from the case of b_1 and b_2, the "GSO phase" of this set must be [32]:

$$\delta_{b_3} = -1 \,, \tag{2.6}$$

(we recall that $\delta_{b_1} = \delta_{b_2} = 1$ and $\delta_S = \delta_{\bar{S}} = -1$). This condition projects out all the states of the type $\phi^L \otimes \phi^R$, for whatever indices and $\phi \in \{\psi, \chi, y, \omega\}$, i.e. all the states of the untwisted sector. The moduli "T" and "U" are now "twisted", and the only massless states come from the twisted sectors. The projection coefficients of the fermionic construction are

[31] On the heterotic side, matter and gauge sectors are exchanged by an exchange of the twisted and the untwisted sectors. This corresponds to an inversion of the world-sheet parameter τ: $\tau \to -1/\tau$. This parameter is integrated out, and it never appears explicitly in the effective theory. On the other hand, we have seen that the world-sheet coordinates are roughly "identified" with the two longitudinal coordinates of the light-cone gauge. Any trace of the moduli of this symmetry is therefore hidden by the gauge fixing.

[32] We refer the reader to [43] for an explanation of this coefficient and its role.

given in the following table:

	F	S	$\bar{S}$	b_1	b_2	b_3
F	1	-1	-1	1	1	1
S	-1	1	1	-1	-1	-1
$\bar{S}$	-1	1	1	-1	-1	-1
b_1	1	1	1	1	1	1
b_2	1	1	1	1	1	1
b_3	1	1	1	1	1	1

(2.7)

together with the conditions: $\delta_S = \delta_{\bar{S}} = \delta_{b_3} = -1$, $\delta_\phi = \delta_{b_1} = \delta_{b_2} = 1$. Observe that, with this choice, b_3, although a type II symmetric twist as b_1 and b_2, projects the states with the same phase as a heterotic Z_2 orbifold projection, as we precisely wanted. Notice also that, differently from how it appears on the heterotic side, the projection introduced by b_3 is not exactly symmetrical to the one introduced by b_2. For instance, it seems that it would project out all the T and U fields even when acting alone, i.e. before the introduction of b_2. This impression is however misleading, in that it neglects that, as we have seen, from the point of view of this two-dimensional compactification, these fields are no more moduli of a torus, but have a more complicate expression as functions also of the former coupling coordinate, here "embedded" in the further, fourth torus. And indeed, if we want to introduce the "planes" as in Ref. [14, 17] in order to lower the rank of the twisted sectors, the sets which introduce separate boundary conditions for the coordinates must be defined in order to include more than one bosonic coordinate. Namely, they must contain also the "coupling plane". In the $\mathcal{N}_4 = 2$ model constructed with just $\{b_3, b_1\}$ (or $\{b_3, b_2\}$) the moduli T^i, U^i are no more built from the states:

$$\delta_{ij}\, x_i \bar{x}_j |0> \tag{2.8}$$

but as combinations of states of the type:

$$x_i \bar{x}_j |0> \qquad i \neq j\,,\ \{i,j\} \in (\{3,4\}, \{5,6\}, \{7,8\}) \ \cup \{11,12\}\,. \tag{2.9}$$

The partition function of this orbifold is given by the integral over the modular parameter τ, with modular-invariant measure $(\,\mathrm{Im}\ \tau)^{-2} d\tau d\bar{\tau}$, of:

$$Z^{\mathrm{string}} = \left(\frac{1}{2}\right)^3 \sum_{(H_1,G_1,H_2,G_2,H_3,G_3)} Z^F_{\mathrm{L}}\ Z^F_{\mathrm{R}} \sum_{(\gamma,\delta)} Z_{8,8}\begin{bmatrix}\gamma\\ \delta\end{bmatrix}\,, \tag{2.10}$$

where $Z^F_{L,R}$ contain the contribution of the world-sheet fields $\psi^{L,R}_\mu$, $\chi^{L,R}_a$ (the sets S and $\bar{S}$); $Z_{8,8}$ substitutes what in four dimensional constructions is $Z_{6,6}$, the $c = (6,6)$ internal space. Now this space spans all bosonic degrees of freedom and has $c = (8,8)$, corresponding to the fields $\omega^{L,R}_I$, $y^{L,R}_I$, $I = 1, \ldots, 8$. Notice that we don't have now the factor $1/(\,\mathrm{Im}\,\tau|\eta(\tau)|^4)$, the contribution of the space-time transverse bosonic degrees of freedom, now accounted in

$Z_{8,8}$. We have:

$$Z_{\rm L}^F = \frac{1}{2}\sum_{(a,b)} \frac{{\rm e}^{i\pi\varphi_{\rm L}\left(a,b,\vec{H},\vec{G}\right)}}{\eta^4} \vartheta\begin{bmatrix} a+H_3 \\ b+G_3 \end{bmatrix} \vartheta\begin{bmatrix} a+H_2-H_3 \\ b+G_2-G_3 \end{bmatrix} \vartheta\begin{bmatrix} a+H_1 \\ b+G_1 \end{bmatrix} \vartheta\begin{bmatrix} a-H_1-H_2 \\ b-G_1-G_2 \end{bmatrix} , \tag{2.11}$$

$$Z_{\rm R}^F = \frac{1}{2}\sum_{(\bar{a},\bar{b})} \frac{{\rm e}^{i\pi\varphi_{\rm R}\left(\bar{a},\bar{b},\vec{H},\vec{G}\right)}}{\bar{\eta}^4} \vartheta\begin{bmatrix} \bar{a}+H_3 \\ \bar{b}+G_3 \end{bmatrix} \vartheta\begin{bmatrix} \bar{a}+H_1-H_3 \\ \bar{b}+G_1-G_3 \end{bmatrix} \vartheta\begin{bmatrix} \bar{a}+H_2 \\ \bar{b}+G_2 \end{bmatrix} \vartheta\begin{bmatrix} \bar{a}-H_1-H_2 \\ \bar{b}-G_1-G_2 \end{bmatrix} , \tag{2.12}$$

with:

$$\varphi_{\rm L} = a+b+ab , \tag{2.13}$$

$$\varphi_{\rm R} = \bar{a}+\bar{b}+\bar{a}\bar{b} . \tag{2.14}$$

The contribution of the compact bosons is:

$$\begin{aligned} Z_{8,8}\begin{bmatrix} \gamma \\ \delta \end{bmatrix} &= {\rm e}^{i\pi(H_3+G_3+H_3G_3)} \\ &\times \frac{1}{|\eta|^4}\left|\vartheta\begin{bmatrix} \gamma \\ \delta \end{bmatrix}\vartheta\begin{bmatrix} \gamma+H_3 \\ \delta+G_3 \end{bmatrix}\right|^2 \\ &\times \frac{1}{|\eta|^4}\left|\vartheta\begin{bmatrix} \gamma \\ \delta \end{bmatrix}\vartheta\begin{bmatrix} \gamma+H_2+H_3 \\ \delta+G_2+G_3 \end{bmatrix}\right|^2 \\ &\times \frac{1}{|\eta|^4}\left|\vartheta\begin{bmatrix} \gamma \\ \delta \end{bmatrix}\vartheta\begin{bmatrix} \gamma+H_1 \\ \delta+G_1 \end{bmatrix}\right|^2 \\ &\times \frac{1}{|\eta|^4}\left|\vartheta\begin{bmatrix} \gamma \\ \delta \end{bmatrix}\vartheta\begin{bmatrix} \gamma+H_1+H_2 \\ \delta+G_1+G_2 \end{bmatrix}\right|^2 . \end{aligned} \tag{2.15}$$

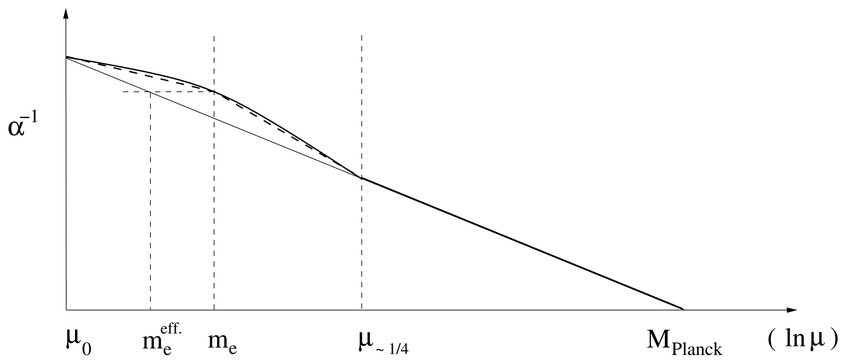
The pairs (a,b) and $(\bar{a},\bar{b})$ specify the boundary conditions, in the directions $\mathbf{1}$ and τ of the world-sheet torus, of the sets S and $\bar{S}$, while (γ,δ) refer to the set of all fermionized bosons; (H_1,G_1), (H_2,G_2) and (H_3,G_3) refer to the sets b_1, b_2 and b_3. Notice the presence of the phase ${\rm e}^{i\pi(H_3+G_3+H_3G_3)}$, corresponding to the choice $\delta_{b_3}=-1$.

In this model there are nine massless sectors, corresponding to the previous b_1, b_2, b_1b_2, the new ones, b_3, b_3b_1, Fb_3b_2, $b_3b_1b_2$, $S\bar{S}b_3b_2$, and the $S\bar{S}$ sector. Only three sectors have a perturbative dual on the heterotic side, and correspond to a tern generated by a pair of intersecting projections. Here $b_3 \cap b_1 \neq \emptyset$ and $b_2 \cap b_1 \neq \emptyset$, while $b_3 \cap b_2 = \emptyset$, therefore the pair is either $\{b_3,b_1\}$ or $\{b_2,b_1\}$. On the sets generated by one of these pairs, the third independent projection doesn't impose any further constraint. The third projection is already "built-in" by construction in the heterotic string, which starts with half the maximal supersymmetry of the type II string. Therefore, apart from the supersymmetry reduction, from the heterotic point of view the further projection triplicates the structure of the $\mathcal{N}_4=2$ model. However, on the type II side, where we have access to all the sectors, we can see that some of the sectors hidden for the heterotic string are not supersymmetric: owing to the δ_{b_3} GSO torsion, the $S\bar{S}$ states are here supersymmetric to nothing, and the same is true for the states of the Fb_3b_2 and $S\bar{S}b_3b_2$ sectors: their superpartners are massive. This is a representation in terms of free fermions of what more generally is a mass shift (see Ref. [14] for a discussion of the translation of the fermionic language in terms of orbifold operations).

With different choices of the relative GSO projections of one sector to the other one, the coefficients $(b_3|b_j)$ in table 2.7, we obtain mirror configurations in which supersymmetry is broken in a different way: a negative projection of b_3 to b_1 and b_2 implies that all the twisted sectors are projected out. Some of them, not as a consequence of a shift, but due to incompatibility of the selected chiralities of the spinors of the twisted sectors. It seems therefore that the model is empty unless the S and $\bar{S}$ projections are removed from the definition of the basis: only the pure Ramond-Ramond sector survives (the projections $(b_3|S)$ and $(b_3|\bar{S})$ remain unchanged). These mirror models seem to exist only at a "delta-function" point in the string moduli space.

C Local correction to effective beta-functions

The running of the electromagnetic and weak couplings in the representation in which they are going to be compared with experimental data is logarithmic, with a slope determined by an effective beta-function coefficient. However, as discussed in section 5.4, around the scale $\sim m_e$, the volumes of the matter phase space are expanded (or, logarithmically, shifted), in such a way that for instance the electromagnetic coupling at the scale m_e (i.e. the fine structure constant) effectively corresponds to the value of the coupling *without correction* at a run-back scale, $m_e^{\text{eff.}}$. The amount of running-back in the scale of the logarithmic effective coupling is equivalent to the amount of the forward shift in the logarithmic representation of the volumes of particles in the phase space. If volumes get multiplied by a factor, their logarithm gets shifted, and so gets shifted back the scale at which the coupling in its logarithmic representation is effectively evaluated. This deviation can be considered as a perturbation of the logarithmic running, that we illustrate here. In the figure, μ_0 stays for the starting scale of the running: $\mu_0 = (1/2)\,\mathcal{T}^{-1/2}$, $\mu_{\sim 1/4}$ for the upper end scale of the matter sector, the thick solid line shows the approximate expected behaviour of the inverse coupling α^{-1}, including the correction to the shape, while the thin solid line indicates the original logarithmic behaviour. The dashed segments indicate the linear approximation of the curve we considered in the footnote at page 61 in order to compute the effective weak coupling at the W-boson scale:



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