

# The superstring representation of the universe of codes

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## Abstract

We discuss the continuum field theory limit of the physical scenario described in Ref. [1], the universe arising from the interpretation of the most general collection of logical codes in terms of distributions of units of energy along units of space. This limit leads in a natural way to string theory as the theory which allows to perturbatively parametrize the geometric structures in terms of propagating particles and fields. We discuss some general properties of the spectrum, masses and couplings, the existence of the strong force, with particular attention to the excited states, and the implications for the physics of high energy colliders.

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## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>From combinatorials to strings</b>	<b>2</b>
2.1	The logarithmic map . . . . .	4
2.2	Entropy in the string phase space . . . . .	6
2.3	The scaling of energy . . . . .	7
<b>3</b>	<b>masses</b>	<b>8</b>
<b>4</b>	<b>the strong force</b>	<b>11</b>
<b>5</b>	<b>Resonances</b>	<b>13</b>
5.1	A string path integral . . . . .	13
5.2	Excited proton states . . . . .	16

## 1 Introduction

In Ref. [1] we have presented an updated discussion of a theoretical scenario which can be viewed as a way of ordering the whole of information in its most generic formulation. In this space of logical structures, or strings of information, we have introduced a time ordering using the natural ordering given by the inclusion of sets, and, through the interpretation of logical codes in terms of distributions of energy along a target space, we have shown how this space leads to a universe with the physical and geometrical properties of the universe we live in, with a three-dimensional space governed by a quantum-relativistic physics. The physical universe is given by the superposition of all the configurations, of any space dimensionality, at a given total amount of energy, which plays also the role of time, or age of the universe. Three dimensional space arises as the dominant configuration, while the configurations not contributing to the “classical” part sum up to produce in any observable quantity one can define in the three dimensional space a smearing which corresponds to the Heisenberg uncertainty. The basic expression is the sum over all the possible energy configurations, weighted by their entropy (i.e. the (relative) weight given by the volume of their combinatorial group) in  $\Psi(E)$ , the space of all the configurations (that is, of all the codes, or logical structures) with a fixed total amount of energy,  $E$ :

$$\mathcal{Z}(E) = \sum_{\Psi(E)} e^{S(\Psi(E))}, \quad (1.1)$$

where  $S(\Psi)$  is the entropy of the configuration  $\Psi$  in the phase space  $\{\Psi\}$ , related to the volume of occupation in the phase space,  $W(\Psi)$ , in the usual way:  $S = \log W$ . This sum can be considered as the “partition function”, or the functional generating all the observables, of the theory. The dynamics is intrinsic in 1.1, which means that the time evolution is uniquely given by the entropy-weighted sum: at any time the universe, and therefore also any subregion/subsystem is given by a staple of configurations, weighted by their entropy in the phase space of all the configurations corresponding to the given total energy, or equivalently age, of the universe. By definition any type of “force” or interaction is therefore entropic.

In this work we continue the discussion, and consider the large-energy limit, in which the discrete universe can be approximated by a continuous description of space and its geometry. In this limit, the physical scenario encoded in 1.1 naturally leads to a parametrization in terms of quantum superstrings. The properties of the mapping to this parametrization allow to recognize in the various perturbative string constructions different realizations of subsets of the same scenario, thereby allowing its identification as the “underlying theory”, which therefore in particular encloses also the so-called M-theory, or whatever is the name one wants to give to this no-better-defined theory. We discuss the relation between the non-perturbative formulation, and the representation of space in the perturbative constructions of the string. In particular, the perturbative limit is important because it allows to identify the spectrum of free particles. We discuss the meaning of mass of an elementary particle and field, and the couplings in a scenario in which, as a consequence of 1.1, the dynamics is of entropic type. We discuss in general how these quantities are related to volumes in the phase space, and how they are computed as functions of the age of the Universe. The detailed inspection of the spectrum and the numerical evaluation of masses and couplings is however left to the analysis of Ref. [2], to which we refer the reader for more information. Particular attention is devoted here to the strong interaction in general terms, to the reason and meaning of the existence of a strong force, besides an (electro-)weak one, discussing how its existence is necessarily required and implied by the coupling with gravity. The last part of the work is a general discussion of the phenomenon of resonance in its various aspects, and how it arises as another consequence of the only rationale of the universe in this theoretical framework, the entropy in the phase space of all the configurations. To this regard, we discuss how the entropy-weighted sum 1.1 reduces in the field theory limit to the Feynman Path Integral. The phenomenon of resonance is considered with particular attention to the physics of particle colliders, with a section devoted to the excited states we expect to show up as resonance picks in the proton-antiproton high-energy collisions.

## 2 From combinatorials to strings

As discussed in Ref. [1], the dominant geometry of the universe at energy  $N$  is the one of a three sphere of radius  $\sim N$ . Here the unit of measure can be identified with the Planck scale. In the limit of large  $N$ , this scenario can be approximated by a description in terms of interacting quantum particles and fields, propagating along a time coordinate. Since we start from a description of every observable in terms of geometric distribution of energy,

these particles and fields will not simply move inside a space within a well defined geometry, but will determine themselves the geometry. Namely, we will have a parametrization of the staple of geometries through propagating fields. To this purpose, we need to associate a fiber to any point (= elementary cell of Planck size) of a base, which must correspond to the space, the three dimensional space, because, according to the analysis in section 3 of Ref. [1], dimensions other than three are already taken into account by the fact of working with quantum objects. We have seen there that the universe behaves like a black hole with horizon at radius  $\mathcal{T}$  (where  $\mathcal{T}$  is the continuum limit of  $N$ ), plus “quantum corrections”; in the parametrization in terms of quantum fields the base is therefore holographic. This means that the independent part of the information we want to parametrize is contained in a two-dimensional sphere which would correspond to the horizon of a black hole extended as much as the age/length of the universe,  $\mathcal{T}$ . The amount of energy we must distribute along the fiber is therefore:

$$E_f = k \frac{1}{\mathcal{T}}, \quad (2.1)$$

where  $k$  is an appropriate numerical coefficient. In total we have:

$$E_{\text{tot}} = (\text{volume of base}) \times E_f \propto \mathcal{T}^2 \times \frac{1}{\mathcal{T}} = \mathcal{T}^2. \quad (2.2)$$

Notice that in this framework  $1/\mathcal{T}$  is the ground energy of the “massless” fields, because in the classical limit, the limit in which we neglect quantum corrections to the geometry, the space is compact (radius  $\mathcal{T} \sim N$ ). The minimal momentum is therefore the inverse of the extension of space. Energies above  $k/\mathcal{T}$  are here considered as quantum fluctuations. We incidentally observe that this is also the ground momentum of a string in a compact space, and the fact that it is “anchored” on a two-dimensional space is quite reminiscent of the fact that, in the light-cone gauge of the four-dimensional compactifications of string theory only two transverse coordinates are independent. Indeed, the existence of a minimal distance, the “Planck length”, means that when we want to parametrize this on the continuum we need extended objects. The string is the minimal one, out of which one can also build more extended ones, which indeed turn out to be generated by string theory<sup>1</sup>. With the string, we can “eat” two coordinates of the target space, and go to the so-called light cone gauge, therefore realizing the identification of the base with the two-dimensional surface of the holographic universe in expansion. These facts are therefore related, although understanding how one comes out with two transverse coordinates in a *flat* space requires some intermediate considerations, that we are going to report. In this set up, extended objects are not only the natural implementation of a theory with a built-in minimal length, but also the only possible objects of a quantum scenario. The reason is that, in a quantum theory, having non-extended objects, like point-like massive particles, means that one has black-holes, namely

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<sup>1</sup>There cannot be a consistent quantum theory of non-extended objects with a cut-off on the length, that establishes the existence of a minimal length, because this is like saying that these objects must be extended. Indeed, by considering interactions, and therefore superpositions, of several ones, one can build a momentum spread that leads to a position uncertainty lower than the minimal length:  $\Delta x \sim 1/\Delta p$  (it is essential here that we are speaking of quantum theory, not simply classical theory). With the string, the extension of the object generates a “dual” sector to momenta, the windings, which somehow say how the theory behaves for lengths lower than the minimal one: in its simplest version, it just reproduces the theory above this length.

objects with an extension below the Schwarzschild radius threshold:  $\Delta x \ll E \sim 1/\Delta x$ . However, as discussed in [3] and [1], in our scenario black holes do not exist as localized objects.

## 2.1 The logarithmic map

This “string” scenario is in its ground non-perturbative and in a regime of full interaction. In order to obtain the properties (spectrum, masses, interactions) of the elementary particles we must decouple the theory. This means going to the flat limit of the space, from a sphere to a product of circles. In [4] and [1] the entropy of the three-sphere has been computed to be:

$$S_{(3)} \sim N^2, \quad (2.3)$$

whereas the entropy of the circle is:

$$S_{(1)} \sim \ln N. \quad (2.4)$$

In our case, owing to holography, the “base” of space is a two-sphere, and decoupling the theory implies its transformation into a torus (the product of two circles):

$$S : N^2 \rightarrow 2 \ln N. \quad (2.5)$$

This corresponds to the coordinate transformation  $N \rightarrow \ln N$ , or, in the continuum limit,  $\mathcal{T} \rightarrow \ln \mathcal{T}$ . This procedure introduces the *perturbative* string construction, compactified on circles (toroidal compactification), which turns out to be the realization of this scenario in a *logarithmic picture*, and justifies working with toroidally compactified string orbifolds in order to derive the *spectrum of free particles*.

In the perturbative string limit, holography reflects in the fact that one can go to light-cone gauge. This interpretation is not evident as long as one considers the space-time to be of infinite extension; however, as soon as space is compactified, this property translates into the fact that space is stirred by the expansion of a massless field, whose propagating degrees of freedom are in bijection with the transverse coordinates of the string target space. It is therefore a co-dimension 1 front (horizon-like) which is blown up. This space reduces to just two dimensions upon reduction of the so-called internal coordinates of the string to the Planck scale size. Indeed, as is well known, a consistent string theory can only be constructed by embedding the string in a higher dimensional target space. The number of these dimensions is fixed by the requirements of supersymmetry (basically needed in order to introduce fermions, i.e. in order to implement a relativistic description of space-time) and quantum consistency, and are apparently not related to the dimension (three) of the space we want eventually describe. These two things are however deeply related. Namely, superstring theory is consistent precisely in the right number of dimensions to make of it the theory which implements a description of the universe we are discussing. Indeed, the eventual number of space dimensions of the universe. i.e. three, is automatically fixed as the minimal number of dimensions string theory can be consistently reduced upon compactification, once canonical quantization is imposed. To this regard, we want to show that the “canonical”

form of the Uncertainty Principle, namely the inequality with the appropriate normalization  $\Delta E \Delta t \geq 1/2$ , which in a relativistic context goes together with  $\Delta P \Delta x \geq 1/2$ , implies, and is implied by, only one dimensionality of space, with a well defined geometry. In our combinatorial construction, Ref. [1], section 3, we have seen that we obtain a "classical"  $D = 3$  dimensional space, *plus* the Heisenberg Uncertainty. The dimensionality of space becomes  $D = 3 + 1$  once we implement the "time"  $\mathcal{T} = E_{\text{tot}}$  in a time coordinate suitable for a field theory description. Taking this into account, what we have seen is that:

$$\text{combinatorial scenario} \Rightarrow [D = 3 + 1] \cup [\Delta E \Delta t \geq 1/2] . \quad (2.6)$$

This means also that:

$$\Delta E \Delta t \geq 1/2 \Leftrightarrow D = 3 + 1 . \quad (2.7)$$

Let us suppose in fact by absurd that  $\Delta E \Delta t \geq 1/2 \Leftrightarrow D \neq 3 + 1$ . Then, in the sum of the rests considered to derive the uncertainty (section 3 of Ref. [1]), the ratio between weight of the classical and weights of quantum configurations is different, something that would lead to a different uncertainty. But there is more:  $\Delta E \Delta t \geq 1/2$  not only is uniquely related to the dimensionality, but also to the geometry of space, because geometries different from the sphere have different entropy, and therefore different weight, leading to a different uncertainty. This means that the relation  $\Delta E \Delta t \geq 1/2$  not only fixes dimension and main geometry, but also the spectrum of the theory.

Let us see now how many internal dimensions do we need. We want to describe all the possible perturbations of the geometry of a sphere in three dimensions, as due to fields and particles that propagate in it. Notice that it is not a matter of building a set of fields *framed* in a certain space, i.e. functions of space-time coordinates. It is a matter of promoting the deformations of the geometry themselves to the role of fields. One may think at a description in terms of vector fields. Once provided with a time coordinate, the three-sphere  $\times$  the time coordinate, which can be considered the  $D = 3+1$  "background" space, corresponds to vector fields possessing an  $SO(3, 1)$  symmetry. However, we must have both bosons and fermions. Fermions are needed because we want a quantum relativistic description of fields. It is relativity what leads to the introduction of spinorial representations of space. This does not mean we need bosons and fermions in equal number, nor even that they must have the same mass (implying supersymmetry of the theory): supersymmetry is not a symmetry of the real world (in the sense of an exact symmetry). In terms of spinorial representations,  $SO(3, 1)$  is locally isomorphic to  $SU(2) \times SU(2)$ , a group with 3+3 generators, which, once parametrized in terms of bosonic fields, correspond to a space with six bosonic coordinates. One would like to conclude that, in order to have both a vectorial *and* a spinorial representation of the background space with all its perturbations we need therefore the original 3+1 *plus* 3+3 internal coordinates. With six internal dimensions it seems we are sure that whatever internal configuration can be mapped to a configuration of space-time, allowing for a non-trivial (and complete) mapping between the "fiber" and the "base" space, ensuring to have a non-degenerate and complete description of all the perturbations. Ten is precisely the dimension of any perturbative quantum superstring. There is however one more coordinate, obtained by the "un-freezing" of the gravitational coupling, the unit scale, which is indeed

the coupling of the theory. Perturbatively, this coupling is flattened into a coordinate (it appears explicitly as such in the 11-dimensional supergravity) <sup>2</sup>.

The tight relation we have found between canonical form of the Heisenberg uncertainty and dimensionality of space, together with the absolute generality of the scenario described by 1.1, namely the fact that it considers the collection of all possible configurations, imply also the universality of its translation into the continuum, in terms of string theory, namely the existence of a unique theory underlying all the possible perturbative constructions.

## 2.2 Entropy in the string phase space

In order to reproduce the scenario of 1.1 and therefore be a representation of the same physics, also the string phase space, i.e. the space of all string constructions, must be ordered according to the energy content. In particular, the string target space must be considered always as compact, with the consequence that supersymmetry is always broken. According to the relation 2.2 the time-ordering through energies translates into an ordering through the (average) radius of compactification. It is not so important to define it more precisely, because entropies in the space of all string compactifications are related to the amount of symmetry possessed by the various configurations. Of course the larger is the volume of the target space, the larger is also the continuous group of space symmetry, but what is going to interest us for the identification of the most entropic configurations at any time of the evolution of the universe is the symmetry of the internal space of a string compactification. On the string space, 1.1 becomes:

$$\mathcal{Z}_V = \int_V \mathcal{D}\psi e^{\mathcal{S}(\psi)}, \quad (2.8)$$

where  $\psi$  indicates now a whole, non-perturbative string configuration, and  $V$  is the volume of the target space, intended as “measured” in the duality-invariant Einstein frame. In order to understand what kind of “universe” comes out of all the possible string configurations we must therefore find out which ones correspond to the maximal entropy in the phase space. It turns out that the string construction with the highest entropy is the one with the lowest amount of symmetry, intended both as geometric symmetry of the target space, and symmetry of the spectrum, being these two aspects tightly related. The symmetries of the target space reflect in fact on the entire string spectrum, in the sense that, if different target spaces of a specific string construction have symmetry represented by the groups  $G$  and  $G'$  respectively, such that  $G' = G/H$ , and the initial spectrum has a symmetry  $\tilde{G}$ , the corresponding spectra will have respectively symmetry  $\tilde{G}$  and  $\tilde{G}'$ , such that  $\tilde{G}' = \tilde{G}/\tilde{H}$ , where  $\tilde{H} \cong H$ . We may say that both  $H$  and  $\tilde{H}$  are representations of the same group, that for simplicity we call  $H$  <sup>3</sup>. Let's consider the action of the group  $H$  on an initial string configuration, that we call  $\Psi$ . That means, the action of  $H$  on its target space and on the

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<sup>2</sup>If one wants to keep part of the non-perturbative string description, i.e. with a non-trivial Planck length, one is forced to keep non-trivial part of the coupling even in a perturbative construction. This may lead to some artifacts, that produce the impression, when looking at just a subset of the construction, that the fundamental theory lives in twelve dimensions (See for instance the works on F-theory, first proposed in [5]).

<sup>3</sup>Notice that we are not saying that  $G \cong \tilde{G}$  nor  $G' \cong \tilde{G}'$ !

spectrum. Let us call  $\Psi'$  the configuration obtained by this modding by  $H$ . Elements  $h \in H$  map  $\Psi'$  to  $\Psi'' = h\Psi'$ , physically equivalent to  $\Psi'$ , in the sense that, by construction, there is a one-to-one map between  $\Psi'$  and  $\Psi''$  which simply re-arranges the degrees of freedom. From a physical point of view, there are therefore  $||H||$  ways of realizing this configuration. The occupation in the whole phase space is therefore enhanced by a factor  $||H||$  as compared to the one of  $\Psi$ . By reducing the symmetry of the target space, we have enhanced the possibilities of realizing a configuration in equivalent ways in the string phase space. In this way, we see that, starting from the most symmetric configuration, perturbatively realized on a product of tori, we obtain the most entropic configuration as the one in which the initial symmetry is reduced to the minimal possible one. As one could expect from the considerations expressed above, it turns out that in this configuration all the coordinates of the string target space are twisted, except, in the light cone gauge, from two transverse, corresponding to the “front” of an expanding universe with three space dimensions (see Ref. [2] for a detailed derivation of this result).

### 2.3 The scaling of energy

Let us now see how does the fiber look in the perturbative string construction. Through the logarithmic map of the coordinates the amount of energy on the fiber is mapped as:

$$E_f = \frac{k}{\mathcal{T}} \xrightarrow{\log} \log \mathcal{T} + \log k. \quad (2.9)$$

The first term on the r.h.s. is the contribution of the zero modes (what comes from the regularization of the target space, and is usually quoted as a “ $\log \mu$ ” term), whereas the second term is the contribution of the internal space of the string. For a compactification in which the entire internal space is twisted <sup>4</sup>, and supersymmetry is broken, this contribution is of order one. It may seem strange that what one writes as coordinates of space-time in the target space of a perturbative string construction are indeed the logarithm of the true, physical coordinates. Usually, this is what one would expect just for the coupling. The reason is that in usual field theory the interacting fields are *framed* in a space-time; here they *are* the space-time coordinates themselves, and the coupling is the scale of the geometry of this space. This property has important consequences for what matters the relation between what one computes in whatever perturbative string vacuum, and the corresponding physical quantity observable in the universe. For instance, let us consider the cosmological constant generated by the breaking of supersymmetry. This is related to the vacuum energy, and it would seem obvious that a breaking of supersymmetry at the Planck scale (here the unit scale) leads to a cosmological constant of order one. However, from the expression 2.9 one can clearly see that an additive contribution to the energy, in this case of order one, from a physical point of view is a multiplicative renormalization. Indeed, the Planck scale is not correctly represented

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<sup>4</sup>We make here an extensive use of the language and properties of string orbifolds, but the same considerations apply also to other types of compactifications: in general the term  $\log k$  is the volume of the internal space. Using the language of orbifolds is here justified by the fact that of this type turns out to be the structure of the most entropic string vacuum. In particular, the radius of the internal space is of order 1.

in the perturbative string vacuum. In the scenario implied by 1.1 supersymmetry is indeed broken at the Planck scale, and the cosmological term is of order  $1/\mathcal{T}^2$ . This is true also in the string representation, once the artifacts of the bad representation introduced in the perturbative construction are taken into account. From a formal point of view, this is done by changing the normalization of the string amplitudes, as it was proposed in Refs. [2]: if one considers that, out of the approximation of the perturbative construction, the theory is defined on a compact space, the vertex operators are not to be normalized by the volume of space, i.e. the volume of the group of translations in the four-dimensional space time. There is in fact no more symmetry under translations, and therefore no over-counting along the orbit of this group, a displacement in space or time representing now an evolution of the universe to a different age. As a consequence, one does not compute anymore densities but global quantities that, in order to be correctly inserted in an effective action, must be divided by an appropriate volume factor of the space-time. A quantity of order one, such as the vacuum energy in the case of supersymmetry broken at the Planck scale, must then be divided by the volume of the base, picking a factor  $1/\mathcal{T}^2$ , the right factor to give the correct size of the cosmological term, as well as the energy density, at present time<sup>5</sup>. Considering string theory as *defined* on a compact space, and viewing infinitely extended space only as a limiting case of a compact space, entails therefore a *deep change of perspective*, full of consequences for the interpretation of things that we compute in string theory.

### 3 masses

In this scenario, masses are energy clusters that propagate at a speed lower than the one of expansion of the universe itself, and can therefore be localized in some way. Like the spectrum of elementary fields and particles, also their masses must be explored in the representation in

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<sup>5</sup>The reason why in the traditional approach string computations produce densities, to be compared with the integrand appearing in the effective action, lies in the fact that space-time is assumed to be infinitely extended. In an infinitely extended space-time, there is a “gauge” freedom corresponding to the invariance under space-time translations. In any calculation there is therefore a redundancy, related to the fact that any quantity computed at the point “ $\vec{x}$ ” is the same as at the point “ $\vec{x} + \vec{a}$ ”. In order to get rid of the “over-counting” due to this symmetry, one normalizes the computations by “fixing the gauge”, i.e. dividing by the volume of the “orbit” of this symmetry  $\equiv$  the volume of the space-time itself. Actually, since it is not possible to perform computations with a strictly infinite space-time, multiplying and dividing by infinity being a meaningless operation, the result is normally obtained through a procedure of “regularization” of the infinity: one works with a space-time of volume  $V$ , supposed to be very big but anyway finite, and then takes the limit  $V \rightarrow \infty$ . In this kind of regularization, the volume of the space of translations is assumed to be  $V$ , and it is precisely the fact of dividing by  $V$  what at the end tells us that we have computed a density. In any such computation this normalization is implicitly assumed. In our case however, there is never invariance under translations: a translation of a point  $\vec{x} \rightarrow \vec{x} + \vec{a}$  is not a symmetry, being the boundary of space fixed. On the other hand, a “translation” of the boundary is an expansion of the volume and corresponds to an evolution of the universe, it is not a symmetry of the present-day effective theory. In our framework, the volume of the group of translations is not  $V$ . Simply, this symmetry does not exist at all. There is therefore no over-counting, and what we compute is not a density, but a global value. In our case, compactification of space to a finite volume is not a computational trick as in ordinary regularization of amplitudes, it is a physical condition. In our interpretation of string coordinates, there is therefore no “good” limit  $V \rightarrow \infty$ , if for “ $\infty$ ” one intends the ordinary situation in which there is invariance under translations. In our case, this symmetry appears only strictly at that limit, a point which falls out of the domain of our theory.

which these degrees of freedom show up, namely, in the string representation. Masses appear as the lowest momentum of a given particle, and are related to the scale of the universe, which, we recall it, at any time corresponds to a space-time of finite extension. Indeed, since space is of finite extension, and, in the limit in which free fields and particles show up, “compact” (a torus), the lowest momentum is simply the inverse of the radius of the classical universe,  $1/\mathcal{T}$ , and corresponds to the typical lowest energy of a *massless* field living in a box of size  $\mathcal{T}$  with periodic boundary conditions. This is also the minimal energy of a photon (and in fact it does not make any sense to think of probing an energy corresponding to a wavelength longer than the universe itself up to the horizon of observation, namely longer than a light-path from the big-bang to us). Of course, in the limit in which this space is considered of infinite extension, as are all the current string constructions, this appears as a true massless field. Massive fields are generated via symmetry reduction. A reduction of the symmetry on the fiber leads to a higher concentration of energy, and therefore also of ground energy, producing true massive particles and fields. Let us consider we start with a massless field (i.e. with “mass”  $1/\mathcal{T}$ ) with multiplicity  $k$ , and therefore also symmetry  $G_0$  with volume  $\|G_0\| = k$ . Massive states correspond to distributions of the same amount of total energy along the fiber with a lower symmetry, so that:

$$\frac{m_i}{m_0} = \frac{\|G_0\|}{\|G_i\|}, \quad (3.1)$$

$m_0$  being here the lowest momentum, indeed the minimal energy of massless states. Roughly speaking, this relation says that we can have a certain number  $p$  of states with a certain mass  $m_p$ , or a number  $p/2$  of states with mass  $2m_p$ , and so on. The configuration with the lowest symmetry is the one that produces the free state with highest mass. The string vacuum with lowest symmetry is indeed a superposition (a staple) of configurations, such that the state of lowest mass appears once as “stand-alone” state,  $(\|G_0\| - \|G_1\|)$  times stapled to the state of mass  $m_1$ , which in turn appears  $(\|G_1\| - \|G_2\|)$  times in the staple forming the state with mass  $m_2$  (and therefore contains also the state of mass  $m_0$ ) and so on, in a pyramidal sequence. As a consequence, ratios of masses are given as ratios of volumes in the phase space of propagating degrees of freedom. Owing to the artifacts of the logarithmic representation, what appear as rigid ratios translate into ratios of exponents of the age of the universe, so that the physical masses are given as a sequence of the type:

$$\frac{m_i}{m_0} = \frac{1}{\mathcal{T}^{\frac{\|G_i\|}{\|G_0\|}}}. \quad (3.2)$$

As one can see, heavier masses are not the same as higher momentum excitations, which are multiples of a fundamental one, like the higher frequency modes of a string. In the series of elementary masses, there is no particle with a mass given exactly as a multiple of another one. Therefore, a transition from a particle of higher mass to a (set of) lower mass particles, that is, a decay, always entails an energy gap which goes into kinetic energy. This is precisely what, according to our scenario, makes such a transition physically favoured as compared to its non-occurring: it produces a higher spread of energy along space, thereby increasing the symmetry of the geometry, and therefore the overall entropy of the universe (see Ref. [1])

for the relation between entropy and symmetry of the geometry). The “coupling” of the interaction depends therefore on the amount of momentum/energy space which is made free by the transition. We define here the couplings as ratios of weights on the fiber, i.e. of volumes of symmetry groups; they can therefore be translated into ratios of masses. The amplitude of transition from the particle  $i$  to the particle  $j$  with  $m_j < m_i$  is given by:

$$\alpha_{ij} = \frac{m_j}{m_i}, \quad m_j < m_i. \quad (3.3)$$

This simply expresses the fact that the particle  $i$  “contains” the particle  $j$  in its phase space, and the higher is the ratio  $m_i/m_j$ , the higher is the number of type- $j$  particles in the phase space of  $i$ , namely the higher is the appearance of  $i$  in the form of the particle  $j$ <sup>6</sup>. This at least for a transition not involving a boson exchange. It can be called a “rigid” transition. Of this type are transitions like the CP violating effects, that we consider in detail in [7]. As we show there, only in first approximation, and up to a very limited extent, these phenomena can be parametrized within a traditional gauge field theory approach: the incapacity of correctly accounting for the amount of CP violation in the  $D$ -mesons system, as well as the failure in predicting, even approximately, the baryonic asymmetry, are signals of the problem.

Due to its being the superposition of all possible configurations, in the universe of this scenario all symmetries are broken, and this reflects also in the fact that there are no elementary states with the same mass. What survives the breaking is the  $U(1)$  (gauge) symmetry corresponding to the photon. From a technical point of view, its survival is related to the basic representation of matter as complex fields, a structure explicitly preserved in any superstring construction. From a physical point of view, this construction is precisely tuned in a way to preserve the spinorial character of the fundamental description of space-time, as required by the combination of quantum mechanics and relativity. For transitions involving the exchange of bosons other than the photon (weak decays), the coupling and the mass of gauge bosons is still related to a ratio of volumes, in this case through a composite relation:

$$\alpha_{ij} = \frac{m_j}{\alpha_{iW}^{-1} M_W}, \quad (3.4)$$

where

$$\alpha_{iW} = \frac{m_i}{M_W}. \quad (3.5)$$

This transition has in fact to be considered a composite one, as if it was made of two “rigid” transitions, one from particle  $i$  to  $W$ , and then the other from  $W$  to particle  $j$ . The amplitude is therefore the ratio of the volume of particle  $j$  to the effective volume of the boson  $W$ , i.e. the fraction of the volume of  $W$  projecting on the particle  $i$ , and therefore the volume of  $W$  projecting on both the particles, common to both the transitions. This can be rewritten as:

$$\alpha_{ij} = \frac{m_i m_j}{M_W^2}. \quad (3.6)$$

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<sup>6</sup>Although expressed in an additive form because of the logarithmic representation, the relation 3.3 is the one which is found in semi-freely acting orbifold constructions in which the rank of the gauge group is reduced by raising the rank of the representation, like those considered in Ref. [6].

Notice that this can be viewed also as the averaged coupling of the boson to the pair  $ij$ . We refer the reader to Ref. [2] for a detailed computation, which shows that indeed the so defined masses and couplings reproduce with astounding accuracy the experimental ones, as functions of the age of the universe.

#### 4 the strong force

The couplings we have just defined correspond to the “weak” interactions of the theory. However, together with the gravitational interaction, which in our framework is by definition the fundamental one, the one setting the unit scale, they do not exhaust all the types of interaction. Let us consider again the decay of particles. We have said that it is entropically favoured. However, in itself it leads to a universe made out of just the lightest particles (the first generation of neutrino, electron and quarks). It would seem all fine, but a universe made just out of these free particles breaks the geometric interpretation of the scenario itself. This is to be expected, because free particles are obtained precisely in the limit of decoupling the theory, and in particular of flattening the geometry of space. Reintroducing gravitation leads necessarily to the strong coupling of the theory. In order to see how this precisely works, let us consider what is expected to be the “mean mass”, namely the typical mass eigenvalue of this space. This must correspond to the typical ground momentum, given as the inverse of the mean radius of space. Indeed, since we are talking of elementary masses, the masses of degrees of freedom which appear in the string representation, this radius is not the radius of the three-dimensional sphere, but the one of the full string space. According to our considerations about the number of internal dimensions, it turns out that we have a kind of ellipsoid with 10 space dimensions (eleven-dimensional space-time), of which 3 are extended up to  $\mathcal{T}$ , whereas the remnant 7 are frozen at the Planck length, the unit scale. The corresponding mass is therefore:

$$\langle m \rangle = \frac{1}{2} \left[ \sqrt[10]{\left( \prod_i^{10} R_i = \mathcal{T}^3 \times 1^7 \right)} \right]^{-1} = \frac{1}{\mathcal{T}^{3/10}}. \quad (4.1)$$

This is the mass scale of stable matter, neutral for all the interactions (it is the mass of a hypothetical particle of which our universe would be made if it had only gravitational interactions). As discussed in Ref. [2], this corresponds to the mass of the system neutron+proton+electron+neutrino plus their conjugates, therefore more or less four times the neutron mass  $m_n$ , producing the relation:

$$m_n = \frac{1}{8} \mathcal{T}^{-3/10}, \quad (4.2)$$

which can be used in order to derive the precise value of the age of the universe to be inserted in all the other mass and coupling expressions. The neutron mass turns out to be higher than the mass of the bare quarks of lowest mass. This means that the only process of weak decay alone leads to stable matter of weight too low to ensure the existence of a geometric scenario, implying that there must be another type of force at work, stronger than the gravitational

one, which counterbalances the electro-weak one. It is the geometry, based on the Planck scale, what requires the existence of both types of interactions! At the string level, this is realized through the existence of T-duality, the stringy way of implementing the existence of a minimal length, ensuring thereby that the string is consistently an extended object. Since in the string realization the coupling of the theory too is a coordinate, T-duality results in a so-called S-duality, namely the strong-weak duality. Much like T-duality, also S-duality is eventually broken in the configuration of highest entropy. Nevertheless, it does not completely disappear: simply, strongly and weakly coupled sectors are not perfectly symmetrical to each other. A consequence of T- or S-duality is also that there is no perturbative string realization in which all the states and their interactions are visible. The string compactified on circles, as is our case, has momenta and windings, and one cannot wash out the ones or the other: any perturbative realization is based on a choice of limiting procedure, in which one decides which ones have to appear and which of the two (momenta or coupling) must be truncated out. In infinite space-time one could think to take a freely-acting orbifold and keep just the ones or the other, thereby realizing perturbatively the full theory. But in this scenario, space is compact, and there is always a part of the theory which is simply “hidden”.

Let us now see how the strong force precisely acts on the mass values. We try to derive the exponent expressing the power of  $\mathcal{T}$  corresponding to the mean mass scale (3/10 in our geometric evaluation of above) by simply averaging on the various bare masses of the sequence 3.2, that is, averaging over the groups  $G_i$  (we have one state for each symmetry group). This means taking the average over the projections we have to apply in order to arrive at the string configuration of minimal symmetry. As explained in Ref [2], in the  $Z_2$  orbifold approximation non-vanishing masses are generated by orbifold shifts along the two transverse coordinates of space-time (the base in the language of the previous sections). There is room for two  $Z_2$  shifts which act as 1/2-scale factor projections in the logarithmic picture, therefore all elementary mass scales fall between the 1/2 (square-root) and the 1/4 (fourth root) scale of the universe <sup>7</sup>. The average scale is roughly obtained as:

$$\langle \text{root} \rangle \approx \frac{1}{\Delta x} \int_2^4 \frac{1}{x} dx = \frac{1}{2} \ln 2 \sim 0,34657\dots \quad (4.3)$$

Inserting the value of the age of the universe, in inverse Planck units  $\mathcal{T} \sim 5 \times 10^{60}$  (see Appendix of Ref. [2]), we obtain a mass scale  $\langle m \rangle = \mathcal{T}^{-\langle \text{root} \rangle} \sim 11,2 \text{ MeV}$ , leading to a neutron mass  $m'_n = \frac{1}{8} \langle m \rangle \sim 1,4 \text{ MeV}$ , around 670 times smaller than the actual neutron mass. This is close to the mass scale of the bare quarks of the first family! The strong force acts raising the mean mass scale, because it assigns a larger fraction of the phase space to the quarks as compared to the leptons. We can try to account for this asymmetry by correcting the mean scale by a factor 6/7, obtained by considering that the projection leading to the separation between leptons and quarks, and thereby separating the electron from the lightest quark the up quark, counts as 1/7 of the total group volume (see Ref. [2]), but effectively

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<sup>7</sup>The square root scale is the one of the appearance of masses, and therefore the one of the smallest non-vanishing mass, while with the second shift one obtains the string configuration of minimal symmetry, and therefore the one giving rise to the massive state of highest mass.

produces almost no mass difference  $m_e \sim \mathcal{O}(m_u)$ . In this way we obtain:

$$\langle \text{root} \rangle \rightsquigarrow \sim 0,29706\dots, \quad (4.4)$$

already much closer to the value 0,3 of expression 4.1.

## 5 Resonances

Resonances are a well known effect occurring in physical systems, both at the macroscopic level, for instance in case of momentum transfer between scattering balls or particles, vibrating strings etc..., and at the microscopic level. Of this type are in fact also the absorption of radiation by an atom, or a pick of scattering cross section when a threshold of production of a real particle in the otherwise virtual intermediate channels is opened. In particular this last phenomenon is used as signal of the existence of particles/fields in high energy accelerators. Common to all these phenomena is the energy transfer from a system to another one, when the amount of energy corresponds to a typical emission/absorption band. For what concerns the opening of real channels, the effect is formally parametrized by the (denominator of the) field theory propagator, of the type  $\sim \frac{1}{p^2 - m^2}$  where  $m$  is the mass of the transferred particle or boson, which has a singularity at  $p^2 = m^2$ , leading to a sudden increase of the (integrated over the momenta and mediated) amplitude. The propagator on the other hand shows up as the inverse of the kinetic term of the Lagrangian. In fact, it is already contained in the principle of minimal action, corresponding to the vanishing of the term  $T - V$ , which translates here into (Kinetic Energy)  $-$  (Rest Energy), and as such can be also seen to directly derive from the field theoretical version of the Feynman Path Integral. This phenomenon appears therefore to be correctly implemented in the theory, and not simply “introduced ad hoc”. However, besides the rather refined technical definitions and implementations, the problem of a deeper understanding of resonance is simply translated in understanding why should the evolution of a system be driven by an action principle. In our framework, the entire dynamics is of entropic type, and phenomena do occur simply because they dominate from a simple combinatorial point of view the phase space of all possible configurations. Entropic are not only all forces, but, as we have discussed, the very existence of a three dimensional universe, and its quantum and relativistic nature. We expect therefore that also resonances should find an explanation of this kind. To see that indeed it is so, we first make a digression and show how the sum over configurations weighted by their entropy indeed reduces, in the field theory limit, to the Path Integral.

### 5.1 A string path integral

Any configuration  $\psi_V$  contributing to 2.8 describes in itself a “universe” which, along the set of values of  $V$ , undergoes a pressureless expansion. In this case, the first law of thermodynamics:

$$dQ = dU + PdV, \quad (5.1)$$

specializes to:

$$dQ = dU. \quad (5.2)$$

Plugged in the second law:

$$dS = \frac{dQ}{T}, \quad (5.3)$$

it gives:

$$dS = \frac{dU}{T}. \quad (5.4)$$

Here  $T$  is the temperature of the universe, defined as the ratio of its entropy to its energy. In the case of the configuration of maximal entropy, the universe behaves, from a classical point of view, as an expanding, three-dimensional Schwarzschild black hole, and the temperature is proportional to the inverse of its total energy, or equivalently, its radius:  $T = \hbar c^3 / 8\pi GMk$ , where  $k$  is the Boltzmann constant and  $M$  the mass of the universe, proportional to its age according to  $2GM = \mathcal{T}$ . By substituting entropy by energy and temperature in 2.8 according to 5.4, we get:

$$\mathcal{Z} \equiv \int \mathcal{D}\psi e^{\int \frac{dU}{T}}, \quad (5.5)$$

where  $U \equiv U(\psi(T))$ . If we write the energy in terms of the integral of a space density, and perform a Wick rotation from the real temperature axis to the imaginary one, in order to properly embed the time coordinate in the space-time metric, we obtain:

$$\mathcal{Z} \equiv \int \mathcal{D}\psi e^{i \int d^4x E(x)}. \quad (5.6)$$

Let's now define:

$$\mathcal{S} \equiv \int d^4x E(x). \quad (5.7)$$

Although it doesn't exactly look like,  $\mathcal{S}$  is indeed the Lagrangian Action in the usual sense. The point is that the density  $E(x)$  here is a pure kinetic energy term:  $E(x) \equiv E_k$ . In the definition of the action, we would like to see subtracted a potential term:  $E(x) = E_k - \mathcal{V}$ . However, the  $\mathcal{V}$  term that normally appears in the usual definition of the action, is in this framework a purely effective term, that accounts for the boundary contribution. Let's better explain this point. What one usually has in a quantum action in the Lagrangian formulation, is an integrand:

$$L = E_k - \mathcal{V}, \quad (5.8)$$

where  $E_k$ , the kinetic term, accounts for the propagation of the (massless) fields, and for their interactions. Were the fields to remain massless, this would be all the story. The reason why we usually need to introduce a potential, the  $\mathcal{V}$  term, is that we want to account for masses and the vacuum energy (in other words, the Higgs potential, and the (super)gravity potential). In our scenario, non-vanishing vacuum energy and non-vanishing masses are not produced, as in quantum field theory, through a Higgs mechanism, but arise as momenta of a space of finite extension, acted on by a shift that lifts the zero mode (see Ref. [2]). When we minimize 5.7 through a variation of fields in a finite space-time volume, we get a non-vanishing boundary term due to the non-vanishing of the fields at the horizon of space-time (moreover, we obtain also that energy is not conserved). In a framework in which space-time is considered of infinite extension, as in the traditional field theory, one mimics this term

by introducing a potential term  $\mathcal{V}$ , which has to be introduced and adjusted “ad hoc”, with parameters whose origin remains obscure <sup>8</sup>.

The passage from the entropy sum over configurations to the path integral is not just a matter of mathematical trickery. It involves first of all the *reinterpretation* of amplitudes as probability amplitudes. This is on the other hand implemented in the string construction. But besides this, there is something that may look odd at first sight. In the usual quantum (field) theoretical approach, mean values as computed from the Feynman path integral are in general complex numbers, as implied by the rotation on the complex plane leading to a Minkowskian time,  $1/T \rightarrow it$ . Real (probability) amplitudes are obtained by taking the modulus square of them. This means that what we obtain from 2.8, 5.6 is somehow the square of the traditional path integral. This is related to the fact that, in order to build up the fine inhomogeneities of a vectorial representation of space, as implied by the staple of energy distributions, we resort to a *spinorial* representation of space-time. Roughly speaking, spinors are “square roots” of vectors. Indeed, as discussed in [2] and [8], masses are here originated by a  $Z_2$  orbifold shift on the string space. This shift gives rise to massive particles by pairing left and right moving spinor modes (spinor mass terms in four dimensions are of the type  $m\psi\bar{\psi}$ ). The  $Z_2$  orbifold projection halves the phase space by coupling two parts, and raises the ground momentum. In terms of the weight in the entropy sum, we have at the exponent a pairing/projection  $(S + S)/Z_2$ , what makes clear that the amplitudes of 2.8 are squares of those of the elementary fields (with “weight”  $\exp S$ ). Had we just a vectorial (bosonic) representation of space, this would not occur, because vectorial (spin 1, or scalar, spin 0) mass terms are of the type  $m^2 A^2, m^2 \varphi^2$ . That is, a mass pairs with *one* boson (usually one sees this in terms of dimension of the field). One can see that the effective rest energy term  $E_0$  introduced by the existence of a boundary of space has precisely the right sign to produce the kinetic term of type  $E - E_0$ : an effective action on a compact space with energy term  $E$  is equivalent to an effective action with a lower energy term,  $E - E_0$ , integrated over an infinitely-extended space. Therefore, the entropic approach correctly reproduces the term  $E - E_0$  which, once inverted, gives the singular term of the propagator, leading to resonance.

In our theoretical framework, a resonance occurs whenever the initial energy equals the energy of a state of the theory, because in the space of the configurations of energy distributions there is no distinction between “types” of energy: there is only a staple of ways of

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<sup>8</sup>Here we have another way to see why the cosmological constant, accounting for the “vacuum energy” of the universe, as well as the other two contributions to the energy of the universe, correspond to densities  $\rho_\Lambda, \rho_m, \rho_r$ , whose present values are of the order of the inverse square of the age of the universe  $\mathcal{T}$ :

$$\rho \sim \frac{1}{\mathcal{T}^2}. \quad (5.9)$$

Were these “true” bulk densities, they should scale as the inverse of the space volume,  $\sim 1/\mathcal{T}^3$ . They instead scale not as volume densities but as surface densities: they are boundary terms, and as such they live on a hypersurface of dimension  $d = \dim[\text{space-time}] - 1$ . The Higgs mechanism of field theory itself can here be considered a way of effectively parametrizing the contribution of the boundary to the effective action in a compact space-time. The Higgs mechanism, needed in ordinary field theory on an extended space-time in order to cure the breaking of gauge invariance introduced by mass terms, is somehow the pull-back to the bulk, in terms of a density, i.e. a “field” depending on the point  $\vec{x}$ , of a term which, once integrated, should reproduce the global term produced by the existence of a boundary.

assigning a certain amount of energy with a certain space distribution. Localizing an amount of energy corresponding to the mass of a particles is absolutely equivalent to producing a particle with the same degree of localization, for the simple reason that the concepts of particle or wave or what else belong more to our way of organizing the description of physical phenomena than to the intrinsic essence of physical phenomena in themselves. In this sense, also processes of energy emission and/or absorption in atomic systems are types of resonances, and the smearing of the pick (for instance of absorption) has basically the same origin as the quantum nature of physics itself, namely the fact of being the universe a superposition of configurations. In some sense, the  $1/x^2$  behaviour of the propagator  $1/(p^2 - m^2)$  can be considered as the approximation of an exponential (Gaussian) behaviour:

$$e^{-x^2} - 1 \sim \frac{1}{x^2} + \dots \quad (5.10)$$

An example is the case of the emission of radiation from transitions between atomic energy levels, which has an exponential width, usually formalized in the assumption that a physical photon is a wave-packet of solitonic type, therefore a function of the type of hyperbolic sinus, i.e. with a Gaussian dependence on the energy spread. The Gaussian suppression out of the resonance pick is due to the fact that in the micro-universe corresponding to the experiment, with total energy  $E \sim N$ , configurations corresponding to a different total energy  $n < N$  are suppressed by a factor  $e^{n^2 - N^2}$ , as if they correspond to a micro-universe of lower age  $\mathcal{T}' \sim n < \mathcal{T} \sim N$  (see discussion in Ref. [1] about the weight of configurations at previous age / lower energy).

## 5.2 Excited proton states

Since a lot of attention is focussed today on the physics at LHC, it is interesting to investigate, in the light of our theoretical framework, what are the possible resonances to be expected in high-energy proton-antiproton scatterings. Besides the usual thresholds opened at energies corresponding to the production of real particles, there is another kind of enhanced channels, which can only be understood in the light of the non-perturbative framework we have discussed, and the multiplicative properties of the phase space, as opposed to the usual additive description one gives in the perturbative regime, when particles can with good approximation be considered as “free”. Consider the electric force between two charged particles of elementary integer charge  $e$ . Perturbatively (that is, on the tangent space, i.e. in the logarithmic picture, or perturbative string picture) one has:

$$E_V \sim \frac{e^2}{R^2} \sim \frac{\alpha}{R^2}, \quad (5.11)$$

where for simplicity we have neglected all numerical factors and fundamental constants (which can be considered to be set to one). For a “bound” state the distance  $R$  goes “to zero”, that is, in our physical framework, to the Planck length:  $R \rightarrow 1$ . Therefore, for a state such as a proton-antiproton pair at their collision, that we indicate as  $p\bar{p}$ , the electric potential energy is simply:

$$E_V \sim \alpha. \quad (5.12)$$

The total energy in the rest frame of this state is:

$$E_{p\bar{p}} \sim m_p + m_{\bar{p}} + \alpha. \quad (5.13)$$

Out of the logarithmic picture, namely, on the real physical picture, this sum becomes a multiplication, as can be seen by considering the electric interaction between the  $p\bar{p}$  state and its decay product, i.e. the unbound pair of “free” proton and antiproton, that we indicate as  $p \cup \bar{p}$ . This is similar to the relation 3.4 for mass ratios, in which now  $i$  and  $j$  are the  $p\bar{p}$  and  $p \cup \bar{p}$  states, and instead of  $W$  we have the photon, with  $p_\gamma^2 = (m_{p\bar{p}} - m_{p \cup \bar{p}})^2 \sim m_{p\bar{p}}^2$  substituting  $M_W^2$ . We have therefore:

$$\frac{m_{p \cup \bar{p}}}{m_{p\bar{p}}} = \alpha, \quad (5.14)$$

where  $m_{p \cup \bar{p}} = m_p + m_{\bar{p}}$ . Inserting the value of the electric coupling  $\alpha$  at the quark scale,  $\sim 1/133$ , and the proton mass value  $\sim 938, 2\text{MeV}$ , we obtain  $m_{p\bar{p}} \sim 250\text{GeV}$ . However, this is not the lightest excited state: there is also the possibility of forming  $(pe^-)$  excited states, through  $p\bar{p} \rightarrow (pe^-) e^+\bar{p}$ , and then  $(p\mu)$  excited states via  $p\bar{p} \rightarrow (p\mu) \bar{\mu}\bar{p}$ . In this case, the resonance energy is around  $\sim 124, 7\text{GeV}$  and  $\sim 128\text{GeV}$  respectively<sup>9</sup>. The interactions of these excited states are the same of the non-excited state, in the same way as the excite states composed of quarks interact through their elementary constituents, and therefore they inherit the strength of the couplings. What changes are the volume factors due to a different energy gap between initial state and the masses of the final products. The same type of excited states exists also in the lepton-antilepton scattering. However, in this case the resonances, namely the electron excited states, as a matter of fact superpose to the physical particles, which are almost at the same “distance” in the phase space (see Ref. [2] for a detailed analysis of the mass hierarchy).

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<sup>9</sup>Values obtained, as the previous one, with an effective value of the coupling  $\alpha$  rescaled from the electron scale assuming an effective logarithmic running up to the Planck scale.

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